

Contractionary Interest Rate Cuts

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Interest rates have been declining in the past forty years. Nominal interest rates have remained extremely low in the U.S. after the 2008 financial crisis and became negative since 2014 in the Euro Area. Figure 1 shows the evolution of U.S. Federal Funds rates from 1980 to 2019. Figure A.1 in the Appendix shows interest rates and interest rate cuts in the US and the Euro Area since 1970.¹

The decline in nominal interest rates appears to have a number of causes, for instance, slower productivity growth, aging population in advanced economies, and increased demand for safe assets.¹ The current interest rate environment may limit the ability of central banks to counter future economic slowdowns with conventional short-term interest rate cuts.²

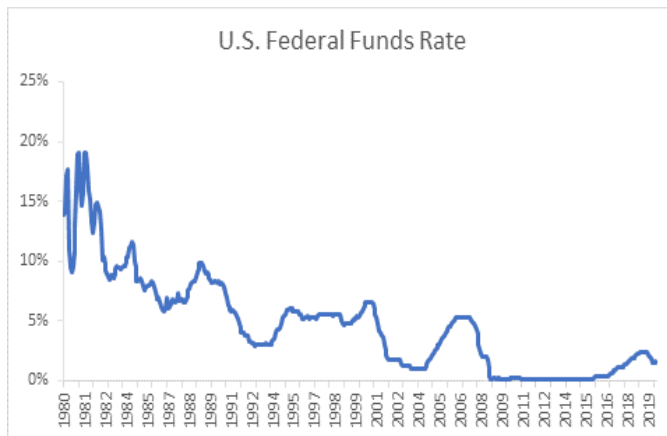


Figure 1: Effective Federal Funds rates from 1980 to present on a monthly basis. Over this time, the Federal Funds rate has peaked at 19.1%, has been as low as .07%, and is currently at 1.55%. Source: Federal Reserve Bank of St. Louis Economic Data and FSF calculations.

In this note, we draw and build on existing research to show that the bank lending channel of monetary policy transmission may break down in the current interest rate environment, where central bank policy rates have remained excessively and persistently low for a long period of time (low-for-long) and have become negative in the Euro Area. More specifically, we show that in the low-for-long or negative rate environment, conventional short-term interest rate cuts can reduce bank lending, increase lending (loan) rates, and may ultimately reduce output.

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¹ Figure A.1 is from [Eggertsson, Juelsrud, Summers, and Wold \(2019\)](#).

² See, for instance, the International Monetary Fund's [World Economic Outlook](#) in January 2020.

We first sketch a banking model to illustrate that capital regulation can induce a lower bound on policy rates below which the bank lending channel may collapse. Next, we highlight some of the empirical evidence on the existence of a lower bound on deposit rates in the Euro Area. Below the deposit rate lower bound, lending rates and volumes seem to stop responding to rate cuts. That is, lending does not increase and loan rates do not drop under rate cuts. We show that interest rate cuts in the negative territory can become contractionary due to capital constraints.

The main contribution of this note is to highlight the mechanism through which capital constraints can make conventional policy rate cuts contractionary in the low-for-long or negative rate environment.ⁱⁱ We also illustrate that the impact of capital constraints may be underestimated in current macroeconomic models.

Policy Rates and the Bank Lending Channel

To show the impact of policy rates on lending, we introduce a simple banking model in partial equilibrium. The stylized model is a variation of the Monti-Klein model and builds upon and captures some of the results of [Brunnermeier and Koby \(2019\)](#) and [Borio, Gambacorta, and Hofmann \(2017\)](#).³ It provides a simple framework through which the interplay of the central bank policy rate, capital regulation, and bank lending can be seen.

As in [Brunnermeier and Koby \(2019\)](#), consider a typical bank from a pool of many identical banks.ⁱⁱⁱ Suppose that at time zero, the bank needs to decide how much deposits D to take, the amounts of loans L to give out, and the amount of investments X to make in a portfolio of securities. The bank chooses D , L , and X at time zero to maximize its net worth in the next period -- a future point in time represented by $T > 0$.

The initial balance sheet identity is $L + X = D$.⁴ As will be shown below, it is useful to think of X as the amount of investment in a portfolio of safe and liquid securities, e.g. government bonds.

The bank is subject to capital regulation in the form of a risk-based capital constraint imposed on loans. Suppose that the risk weight associated with securities holdings is zero. We also assume that $X > 0$. This can be due to liquidity regulation under which the bank needs to hold a minimum amount of safe assets.

³ See [Monti \(1972\)](#) and [Klein \(1971\)](#). Despite its simplicity, insights drawn from the Monte-Klein model have been widely used in practice and can be often confirmed with empirical evidence. Also, see Chapter 3 of [Freixas and Rochet \(2008\)](#) and the references therein.

⁴ We assume zero equity financing; our results remain the same if we add equity financing to the model. [Brunnermeier and Koby \(2019\)](#) results depend in part on banks' initial capitalization. By moving away from equity financing, we will show that the existence of a lower bound on policy rates need not depend on banks' initial capitalization.

That is, the bank chooses D , L , and X at time zero to maximize its net worth N in the next period,

$$\begin{aligned} &\text{maximize } N = r_l L + f(r)X - r_d D \\ &\text{subject to } wL \leq N \end{aligned}$$

In the above formulation, r_l is the interest rate on loans, r_d is the interest rate on deposits, and r represents the central bank policy rate.

In the capital constraint inequality above, w represents loan risk weights times a regulatory capital ratio. The market value of the bond portfolio at time T is represented by $f(r)X$, where $f(r)$ can be viewed as a function of the policy rate whose form depends on the composition and maturity dates of bond contracts in the portfolio. To clarify the purpose of and the intuition behind $f(r)X$, we use the following two simple examples.

Before introducing these examples, we note that in a typical Monti-Klein model, the bank holds X in the central bank reserves at time zero, which earns an interest equal to the central bank policy rate, and so gives rX in the next period. That is, we can set $f(r) = r$ in the classical Monti-Klein model under central bank reserves and in the absence of any fixed-income securities holdings.

Example 1. After choosing X , suppose that the bank invests all X in a zero-coupon bond⁵ that matures at time $S < T$. The value of this investment at time S becomes X . Suppose that the bank then holds all X in central bank reserves earning an interest equal to the central bank policy rate. Let us use continuous compounding in these examples. Then, the value of this hybrid investment at time T becomes $Xe^{r(T-S)}$. Consequently, $f(r) = e^{r(T-S)}$. Note that the derivative of f with respect to the policy rate r , i.e. $f'(r)$, is positive. So, as the central bank cuts rates, the value of this investment decreases. This can be formally seen by taking the derivative of $f(r)X$ with respect to r , which is $f'(r)X \geq 0$.

Example 2. After choosing X , suppose that the bank invests all X in a zero-coupon bond that matures at M with $M > T$. The value of this investment at time T becomes $Xe^{-r(M-T)}$ and so $f(r) = e^{-r(M-T)}$. Note that in this example $f'(r)$ is negative. That is, when the central bank cuts interest rates, the bank gains value from its securities holdings as can also be seen from $f'(r)X \leq 0$.

We now return to the stylized bank net worth maximization model. It is often assumed that D and r_d move in the same direction - D increases (decreases) with r_d . It is also often assumed that r_l and the demand for loan and so L move in the opposite direction.

⁵ A zero-coupon bond that matures at time T pays \$1 to its holder at time T .

Suppose that we want to measure the impact of policy rate on the bank's net worth. Writing N_r for the derivative of N with respect to r , $N_r \equiv dN/dr$, we can measure the impact of r on N by calculating N_r ,

$$N_r = (1 + c)f'(r)X$$

In the above equation⁶, $c \geq 0$, and c depends on the capital constraint in a way that $c(wL - N) = 0$ holds.⁷ The sign of N_r depends in part on asset valuation gains or losses, which in turn depend on maturities of the portfolio compositions.

As in Example 1, we can have $N_r \geq 0$ when asset valuation gains start to diminish under policy rate cuts. This appears to be the case in the low-for-long regime where policy rates have remained excessively low for a long period of time. In the low-for-long environment, legacy fixed-income securities holdings become close to or pass their maturity dates, so potential asset valuation gains from rate cuts can diminish while net interest income decreases.⁸ Indeed, $N_r \geq 0$ has been reported to hold in the post-crisis policy rate environment. This has been documented empirically and analyzed by a number of researchers including [Borio et al. \(2017\)](#).

The Interplay of Monetary Policy and Capital Regulation

To show the impact of the policy rate on lending volumes, we can differentiate L with respect to r , which is represented by L_r . If $L_r \leq 0$, we conclude that decreasing policy rates increase lending. This is how the classical bank lending channel should work in normal times. However, if $L_r \geq 0$, policy rates and lending move in the same direction. We will now illustrate that in the low-for-long environment, because of capital regulation, $wL \leq N$, if policy rates fall below a certain level denoted by \underline{r} , bank lending starts decreasing. That is, if the central bank cuts policy rates below the lower bound \underline{r} , we will have $L_r \geq 0$.

It was shown earlier that $N_r \geq 0$ in the low-for-long environment. That is, cutting rates decreases banks' net worth. In the absence of capital regulation, the optimal solution to our net worth maximization problem gives $r_l = f(r) + e_l$ where $e_l > 0$ is the inverse of the semi-

⁶ N_r is derived using the Envelope Theorem. See, for instance, p. 604 of [Jehle and Reny \(2011\)](#).

⁷ c is the Lagrange multiplier associated with the Lagrangian. That $c \geq 0$ and $c(wL - N) = 0$ hold follows from Kuhn-Tucker conditions.

⁸ It is customary to decompose banks' net worth or profits to net interest income (NII) and non-interest income. For instance, we can also write $N_r = c \frac{d[r_l L + f(r)X - r_d D]}{dr} + \frac{df(r)X}{dr}$. If we view $r_l L + f(r)X - r_d D$ as NII in this equation, N_r is then decomposed to the derivative of NII with respect to r , and the derivative of asset holdings w.r.t. the policy rate. It is well-known ([Borio et al. \(2017\)](#)) that NII has decreased under rate cuts in the low-for-long environment. So, when potential asset valuation gains $f'(r)X \leq 0$ are small compared to $c \frac{d[r_l L + f(r)X - r_d D]}{dr} \geq 0$, we will have $N_r \geq 0$. The basic point here is that empirical evidence indicates $N_r \geq 0$, and the Monti-Klein framework is flexible enough to capture that.

elasticity of the demand for loans.⁹ In the absence of capital regulation, policy rate cuts lower r_l and increase L .

Now, let us focus again on the capital constraint

$$wL \leq N.$$

Cutting rates r increases bank lending L (left side of the inequality), reduces the bank's net worth N (right side of the inequality), and so tightens the capital constraint inequality. At a lower bound \underline{r} , the capital constraint inevitably binds, and we will have $wL = N$. Further interest rate cuts below \underline{r} can reverse the capital constraint inequality leading to $wL > N$ -- where capital rules will be violated. Consequently, at or around \underline{r} , the bank would need to cut its lending and instead invest more in the fixed-income portfolio so the inequality $wL \leq N$ can continue to hold. Put differently, due to capital regulation, we will have $L_r \geq 0$ when rates fall below the policy rate lower bound \underline{r} .^{iv} It is important to note that, at least in theory, \underline{r} need not be negative in the low-for-long environment.

In fact, it is not difficult to see that as capital regulation becomes more restrictive, i.e. as w increases in $wL \leq N$, the lower bound \underline{r} also increases. That is, the bank lending channel can collapse faster under more restrictive capital requirements in the low-for-long environment.

The largest U.S. banks' common equity tier 1 capital has grown from \$477 billion in 2009 to \$811 billion in 2019. Figure 2 reports the aggregate common equity tier 1 capital of Bank of America, BNY Mellon, Citi, Goldman Sachs, JP Morgan, Morgan Stanley, State Street, and Wells Fargo from 2009 to 2019. Clearly, the banking system has become more resilient due to the substantial increase in capital requirements. It is, however, unclear whether the interplay of monetary policy and capital regulation in the current interest rate environment can help central banks counter the next economic downturn with conventional monetary policy tools.

As the policy rate falls below \underline{r} , the impact on the lending rate r_l will also be reversed. That is, for rates above \underline{r} , rate cuts will lower lending rates, i.e. $dr_l/dr \geq 0$. But, for policy rates below \underline{r} , interest rate cuts increase lending rates, $dr_l/dr \leq 0$.¹⁰ To summarize, in the low-for-long environment, a policy rate lower bound comes into existence because of capital constraints. Below the lower bound, interest rate cuts decrease lending volumes and increase lending rates.^v

⁹ Specifically, $e_l = -L/L'$, where L' is the derivative of L with respect to r_l . In the absence of regulatory constraints, we have $L' \leq 0$, as lower interest rates on loans increase the demand for bank loans. Under the capital constraint, the optimal solution of our model gives $r_l = f(r) + e_l + cw/(1+c)$. This is derived using the Lagrange's method.

¹⁰ This can be seen using the chain rule $\frac{dL}{dr} = \frac{dL}{dr_l} \cdot \frac{dr_l}{dr}$. For $r < \underline{r}$, we know that $\frac{dL}{dr} \geq 0$. We also know that in general $\frac{dL}{dr_l} \leq 0$. Consequently, we will have $\frac{dr_l}{dr} \leq 0$ for $r < \underline{r}$.

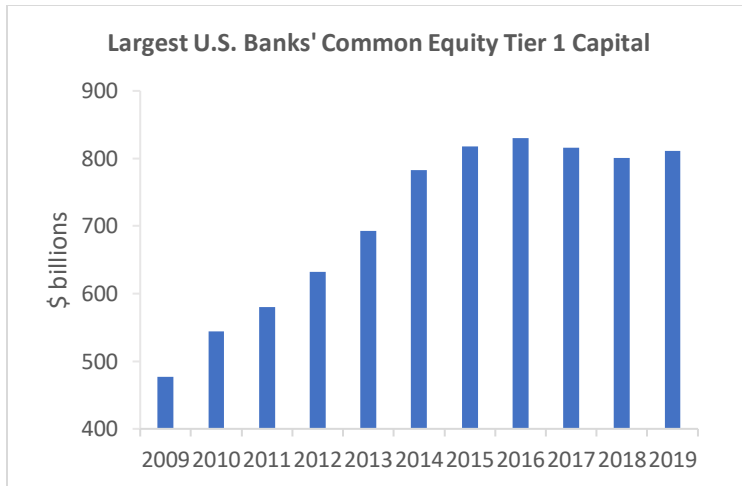


Figure 2: The aggregate common equity tier 1 capital of the largest U.S. banks from 2009 to 2019. Numbers represent end of year data for 2009-2018, and third quarter data for 2019 (latest available). Aggregate common equity tier 1 capital for Forum members has grown from \$477 billion in 2009 to \$811 billion in 2019. Source: Federal Reserve Y-9C.

Deposit Rate Lower Bound

In 2014, a number of central banks reduced their policy rates below zero. When interest rates became negative, the pass-through to deposit rates collapsed to roughly zero. This phenomenon has been documented by a number of researchers, for instance, by [Eggertsson, Juelsrud, Summers, and Wold \(2019\)](#). Figure A.2 in the Appendix shows aggregate deposit rates in Sweden, Germany, the Euro Area, Switzerland, Japan, and Denmark. It can be seen from this Figure that the aggregate deposit rate is below the policy rate and is closely following it when the rate is positive. This relationship breaks down when the policy rate becomes negative. Deposits rates appear to be bounded roughly at zero. This zero-lower bound on deposit rates currently exists in part because depositors have the alternative of holding cash. Future policy reforms can change the storage cost of money and so the lower bound on deposit rate. For instance, in an era with no paper currency ([Rogoff \(2017\)](#)), the storage cost of money and so the deposit rate lower bound will change.

Using bank level datasets for Swedish banks, Eggertsson et al. show that the transmission of policy rates to lending rates is weakened as the policy rate becomes negative. The authors note that under rate cuts in the negative territory between 2014 and 2016, lending rates no longer decreased and lending volumes did not increase particularly for banks with higher deposit shares.

We note that the largest U.S. banks' reliance on deposit financing has increased significantly from 2007 to 2019 as can be seen from Figure 3, which reports the aggregate deposits at Bank of America, BNY Mellon, Citi, Goldman Sachs, JP Morgan, Morgan Stanley, State Street, and Wells Fargo.

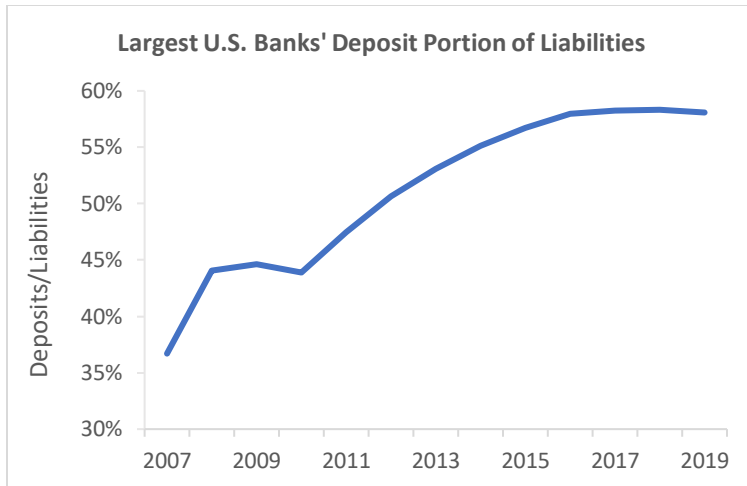


Figure 3: The aggregate deposits of the largest U.S. banks as a portion of aggregate liabilities from 2009 to 2019. Numbers represent end of year data for 2009-2018, and third quarter data for 2019. Aggregate deposits as a portion of liabilities has grown from 37% in 2007 to 58% in 2019. Source: Federal Reserve Y-9C.

Eggertsson et al. show that at the lower bound on the deposit rate, the lending rate and volume stop responding to policy rate cuts. However, there need not be a *reversal* in lending rates and volumes. Under the partial equilibrium banking model of Eggertsson et al., the reversal occurs only under the important assumption that the marginal cost of extending loans decreases with bank net worth or bank profits.^{vi} This assumption, which we will return to shortly, implies that capital constraints can reverse the effects of conventional monetary policy in the negative territory.

In fact, it is not difficult to see from our model that when the deposit rate r_d does not respond to policy rate cuts due to the existence of a deposit rate lower bound, the bank net worth N can decrease under rate cuts, and this can be independent from asset valuation effects discussed in the previous section. Recall that the capital constraint $wL \leq N$ must hold at all times. As we showed earlier, when L increases due to policy rate cuts, and N decreases due to the low-for-long environment or due to the deposit rate lower bound, the direction of the inequality $wL \leq N$ can be reversed. This is exactly when the reversal in the lending rate and volume occurs. That is, even in the absence of being in the low-for-long environment, the combination of rates falling below the deposit rate zero lower bound and stringent capital constraints can guarantee the existence of \underline{r} . Rate cuts below \underline{r} increase lending rates and decrease lending volumes.

Macroeconomic Impact

To quantify the macroeconomic impact of rate cuts in the low-for-long or negative environment, a banking model in partial equilibrium should be embedded in a dynamic general equilibrium model where policy rates can stimulate aggregate demand in the presence of

nominal price rigidities, and where lending rates are endogenously determined by loan supply and demand. This has been done by a number of researchers, for instance, by [Brunnermeier and Koby \(2019\)](#), [Eggertsson et al. \(2019\)](#), and [Kumhof and Wang \(2019\)](#). All results show contractionary effects of low or negative policy rates for output. Low or negative interest rates can also impact aggregate demand through exchange rates, the exchange rate channel has not been considered in this note.¹¹

[Eggertsson et al. \(2019\)](#) show that a policy rate of -.50 percent increases borrowing rates by 15 basis points and reduces output by 7 basis points in Sweden. Brunnermeier and Koby show that the lower bound on the policy rate below which output decreases need not be negative. Below this reversal policy rate, central bank rate cuts become contractionary.

Capital Rules in Macroeconomic Models

We would like to emphasize the role of capital regulation in making policy rate cuts contractionary in the negative or low-for-long environment. Capital or solvency constraints are often captured implicitly, indirectly, endogenously, or in reduced form manners in macro models. In the model of [Eggertsson et al.](#), for instance, banks' capital constraint is implicitly captured through an intermediation cost function. Intermediation costs are increasing in lending. The crucial assumption is that lower bank net worth or profits increase the marginal cost of lending (increase intermediation costs).

In the absence of this assumption, rate cuts cannot be contractionary, (Section 3.1, Section 3.3. and Table 6 of Appendix D). [Eggertsson et al.](#) make this assumption by drawing on the work of [Holmstron and Tirole \(1997\)](#) and [Gertler and Kiyotaki \(2010\)](#) who endogenize financial market frictions by introducing agency problems between borrowers and lenders. In [Holmstron and Tirole](#) equilibrium model of credit, depositors induce solvency conditions on banks, which mechanically resemble exogenous capital constraints. In [Gertler and Kiyotaki \(2010\)](#), an agency problem is induced on banks' ability to obtain external funds, which then mathematically gives rise to a capital constraint essentially similar to $wL \leq N$ in our model.^{vii}

These endogenous capital constraints whose function is similar to regulatory capital, lead to the assumed relationship between banks' net worth and intermediation costs in [Eggertsson et al.](#) Indeed, the model parameter in [Eggertsson et al. \(2019\)](#) that captures the feedback from banks' net worth to intermediation costs and then credit supply specifies the amount of reduction in output due to rate cuts (Figure 16).¹² Higher values of this parameter, which lead to more reductions in output, can imply more stringent capital requirements. In our primitive model,

¹¹ To our knowledge, dynamic general equilibrium models used in the literature to assess the impact of low or negative interest rates have been closed economy models.

¹² The dependence of the intermediation cost function in [Eggertsson et al. \(2019\)](#) on banks' loans and profits has been captured by this form $l^\nu z^{-\iota}$ at any given point in time, with l and z denoting lending volume and bank profit, respectively, and $\nu, \iota \geq 0$. Figure 16 of [Eggertsson et al.](#) show that higher values of ι ultimately lead to more reduction in output. The role of ι can be mechanically viewed as the role of w in our model.

when w increases, the capital constraint $wL \leq N$ becomes more restrictive. More restrictive capital rules make interest rate cuts more contractionary.

In the general equilibrium model of Brunnermeier and Koby, the explicit capital constraint $wL \leq N$ is smoothed and replaced with leverage costs in a reduced form manner (for technical reasons). As also noted by Brunnermeier and Koby, this *smoothing* of the capital constraint causes their model to produce the contractionary effect of rate cuts in the low-for-long environment with time delays (Section 5.4).

The basic point is that capital constraints -- endogenous or exogenous -- link credit supply and bank net worth, and it is important how this link is captured in economic models.

Concluding Remarks

In the low-for-long or negative interest rate environment, conventional monetary policy may break down because of the collapse of the bank lending channel. Short-term interest rate cuts -- which are accommodative and expansionary in normal times -- may become contractionary.

In the low-for-long environment, banks' asset valuation gains under policy rate cuts may diminish, rate cuts can subsequently reduce banks' net worth as net interest income also decreases in this environment. Capital regulation can then induce a lower bound on policy rates. Below this policy rate lower bound, rate cuts may become contractionary. In the negative territory, the deposit rate zero lower bound causes rate cuts to reduce banks' net worth when banks' funding structures rely heavily on deposit financing. Then, capital regulation can similarly induce a lower bound on policy rates.

As capital regulation becomes more restrictive in the low-for-long or negative environment, the policy rate lower bound increases. When the lower bound increases, interest rate cuts can become contractionary faster, and their adverse impact on the economy may become more severe.

Since dynamic general equilibrium models do not currently capture capital rules distinctly, the role of regulatory capital may remain hidden, and its impact on making rate cuts contractionary may be underestimated. Subsequently, the macroeconomic effects of interest rate cuts may be underestimated.

We view our results in the broader context of the impact of *financial conditions* on economic activity. There has been consensus among policy makers and economists on the importance of financial conditions to macroeconomic outcomes since the 2008 financial crisis, (see, for instance, [Adrian, Grinberg, Liang, and Malik \(2018\)](#) and the references therein). It is now also well-known that the behavior of intermediaries is subject to complex, sudden, and nonlinear threshold effects ([Hatzius, Hooper, Mishkin, Schoenholtz, and Watson \(2010\)](#)). Indeed, we have highlighted the threshold-type mechanism through which capital constraints of intermediaries

may reverse the impact of conventional rate cuts -- this mechanism may not be captured appropriately by current macro models.

The interplay of monetary policy and bank regulation that we have touched upon in this note can remain subtle. Mechanisms through which short-term interest rate cuts can become contractionary should be investigated carefully by policymakers, researchers, and regulators.

Appendix

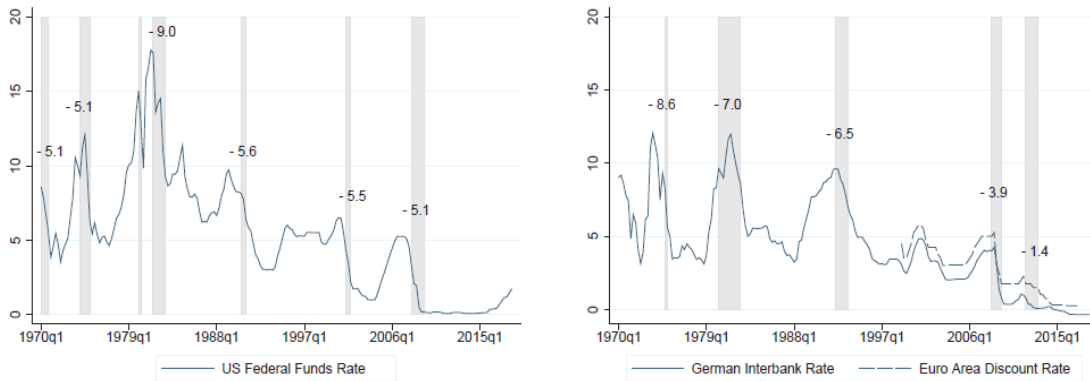
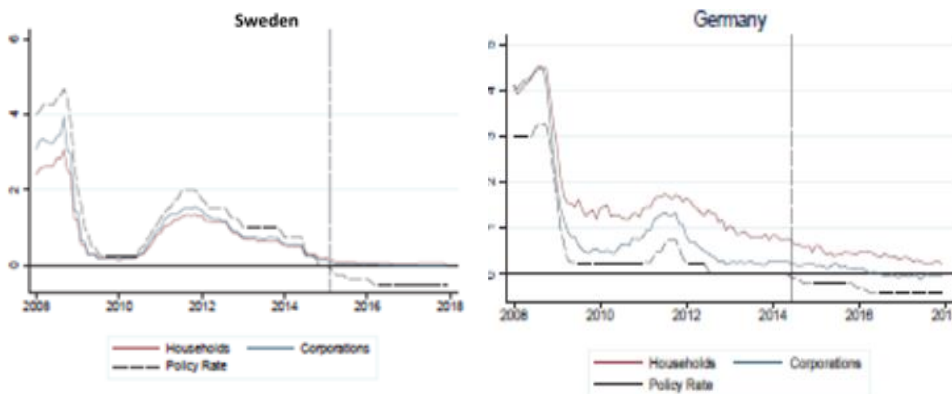


Figure A.1: Interest rate cuts during recessions in the US and Euro Area since 1970. Nominal interest rates are reduced by 5.9 and 5.5 percentage points on average. Policy rates will be in the negative territory with rate cuts of this magnitude. Source: Eggertsson et al. (2019).



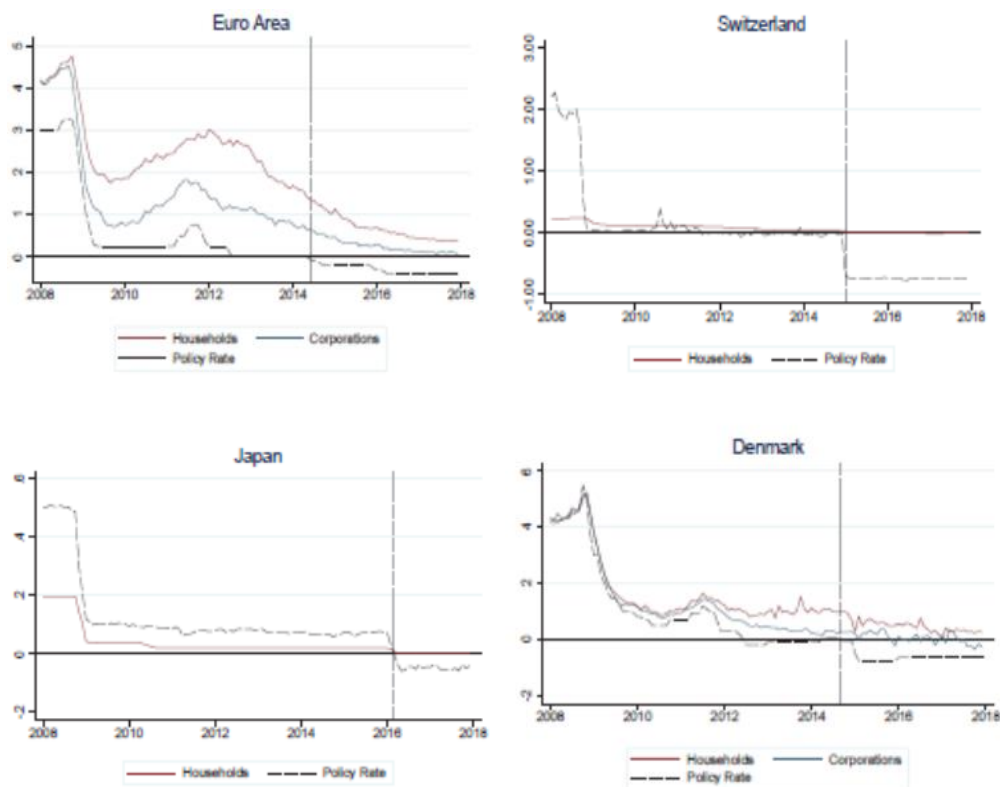


Figure A.2: Aggregate deposits rates in Sweden, Germany, the Euro Area, Switzerland, Japan, and Denmark. The vertical lines mark the month in which policy rates became negative. Source: Figures 3 and 17 of Eggertsson et al. (2019).

ⁱ See [Summers \(2015\)](#), [Del Negro, Giannoni, Giannone, and Tambalotti \(2017\)](#), and the references therein.

ⁱⁱ In this note, we do not discuss *unconventional* monetary policy strategies, e.g. quantitative easing, forward guidance, and yield curve control. We refer the readers to [Bernanke \(2017\)](#), [Bernanke, Kiley, and Roberts \(2019\)](#), and the references therein.

ⁱⁱⁱ We assume perfect competition in the banking sector. This assumption is made for simplicity, it does not change our results and can be relaxed in a straight forward way.

^{iv} [Borio et al. \(2017\)](#) variation of the Monti-Klein model also incorporates capital regulation but does not give the possibility of $L_r \geq 0$ below a policy rate lower bound (because of capital regulation). This is in part because the authors assume constant elasticity of the demand for loans (see their Annex A) to calculate L_r . This assumption need not hold under capital regulation. That is, the work of Borio et al. does not lead to the existence of a policy lower bound induced by capital constraints.

To our knowledge, [Brunnermeier and Koby \(2019\)](#) (B-K) are the first to theoretically show the existence of a lower bound \underline{r} below which $L_r \geq 0$. They call this lower bound the *reversal interest rate*. Our model is different from the B-K model in that we do not need equity financing to show the existence of \underline{r} . The existence of \underline{r} depends in part on banks' initial capitalization in the B-K model. [Borio et al. \(2017\)](#) note

that banks' equity capital is generally less interest rate sensitive than other balance sheet components. Our model captures the market value of securities holdings to some extent while the B-K model does not. This is also a key difference with the B-K model. Brunnermeier and Koby model fixed-income assets similar to central bank reserves.

In the end, the solution to our stylized net worth maximization model is different from the solution of the B-K model. Our primitive model, which is a simple variation of the B-K model, captures banks' fixed-income holdings more realistically (by differentiating them from central bank reserve holdings). Our results do not depend on the sensitivity of equity capital, particularly banks' initial capitalization, to interest rates.

^v In this note, we do not investigate the impact of liquidity regulation on \underline{r} and the bank lending channel. We leave this for future work. Compared to capital rules, it is well-known that assessing the impact of liquidity rules on the financial and economic system is more difficult (Ghamami (2019)). The partial equilibrium model of Brunnermeier and Koby (2019) captures liquidity regulation to some extent and show that, similar to capital regulation, restrictive liquidity rules can also increase the lower bound \underline{r} and so may break down the bank lending channel.

^{vi} Eggertsson et al. (2019) use the banking model of Curdia and Woodford (2011) to show $L_r \geq 0$ can hold in the negative territory under a deposit rate lower bound and under the assumption that the marginal cost of extending loans decreases with bank net worth or bank profits. See also Appendix C, p.48, of Eggertsson et al. (2019) for more on this assumption.

^{vii} More specifically, see the banks' incentive constraint, display (11), in Gertler and Kiyotaki (2010). Solving the Lagrangian of the bank value maximization problem, display (11) gives rise to the incentive constraint (16). Both displays (11) and (16) can be mechanically viewed as variations of our capital constraint $wL \leq N$.

In our primitive model, increasing w makes the capital constraint more stringent, in Gertler and Kiyotaki's model, increasing θ makes their endogenous capital constraint more stringent.