Skin in the Game:
Risk Analysis of Central Counterparties

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Abstract

This paper introduces an *incentive compatibility* framework to analyze agency problems linked to central counterparty (CCP) risk management. We provide a quantitative framework to design CCP skin-in-the-game (SITG). We show that under inadequate SITG levels, members are more exposed to default losses than CCPs. The resulting risk management incentive distortions could be mitigated by using the proposed SITG formulations. Our analysis addresses investor-owned and member-owned CCPs, we also analyze multilayered and *monolayer* default waterfalls. Viewing the total size of SITG as the lower bound on CCP regulatory capital, the framework can be used to improve capital regulation of investor-owned and member-owned CCPs. We also demonstrate that bank capital rules for CCP exposures may underestimate risk. The broader central clearing mandate of U.S. Treasuries may take place under *monolayer* CCPs. These clearinghouses may need to allocate more of their own capital to the default waterfall.

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1 Introduction

The over-the-counter (OTC) derivatives markets reform program launched by the G20 nations after the global financial crisis (GFC) of 2007-2009 has drastically transformed these markets. Central clearing of standardized OTC derivatives has been one of the main components of the reform program (Ghamami and Glasserman, 2017). The Covid-19 crisis revealed that the secondary market for U.S. Treasuries can become dysfunctional in part due to the constraints on the capacity of dealers that intermediate this market (Duffie (2020); Duffie et al. (2023)). The Group of Thirty (G30) proposed a reform program in 2021 to strengthen the resilience of the U.S. Treasury (UST) markets (G30, 2021). Broadening central clearing mandates in government securities markets is one of the main elements of the proposed reform program.

The main potential benefits of central clearing are well-known. Central clearing has the potential to reduce the interconnectedness of the financial system and improve transparency. Central clearing can help mitigate counterparty credit risk[1] through multilateral netting (Duffie and Zhu (2011); Cont and Kokholm (2014); Ghamami and Glasserman (2017); Garratt and Zimmerman (2015)) and when multilateral netting efficiencies dominate bilateral netting, the cost of collateral requirements could be reduced.[2] Central clearing may also reduce the pressure on intermediaries' balance-sheets (Duffie (2020); Baranova et al. (2023)).

CCPs require effective governance, regulatory oversight, and highly robust risk management frameworks. Otherwise, increased use of CCPs may create financial stability risks (Bernanke (2011); Dudley (2014); Tucker (2014)). The failure of a systemically important CCP can be disastrous.[3] The right design and regulation of CCPs continue to generate debate among industry participants, government officials, and the public.[4]

[1]In UST markets, counterparty credit risk materializes mostly in the form of settlement failures (Ingber, 2017). UST market settlement fails rose significantly in March 2020, (Figure 13 in (Duffie, 2020)).

[2]Multilateral netting outperforms bilateral netting under certain conditions. For instance, as the number of asset class specific CCPs increases, and when bilateral netting across an increasing number of asset classes is permitted, bilateral netting can dominate multilateral netting. The overall netting efficiencies achieved by central clearing could also depend on specifics of client clearing models. Research on client clearing is scarce (CPMI (2022); Ghamami et al. (2022)).

[3]CCP failures or near failures are not impossible (Menkveld and Vuillemey, 2021). The most well-known cases are the failure of *the Caisse de liquidation* in Paris in 1974, the Kuala Lumpur Commodity Clearing House in 1983, and the Hong Kong Futures Guarantee Corporation in 1987. Tucker (2014) highlights the impact of the failure of the Hong Kong futures clearing house: "*Basically, Hong Kong's securities markets all stopped, affecting households and firms well beyond the community who had positions in stock-index futures.*"

[4]In 2019 and 2020, major buy-side and sell-side firms called for regulatory action to make clearing houses safer. The industry paper consisted of a number of recommendations.

CCPs rely on their *default waterfall* to manage the pooled counterparty credit risk in centrally cleared markets (Cont (2015); Ghamami and Glasserman (2017); Murphy (2017)). The typical waterfall structure is multilayered and consists of collateral posted by clearing members in the form of *risk sensitive* initial margin (IM) and contributions to a quasi risk-based default (guarantee) fund (DF). CCPs also often make equity capital contributions to the waterfall. These capital contributions are often referred to as skin-in-the-game (SITG). SITG is not risk-based in that it need not correspond to the risk profile of a CCP.

When a member defaults, the CCP first uses the *defaulter-pay resources* to cover losses. Potential remaining losses are *mutualized* among the CCP and surviving members. SITG and surviving members' DF assets can be used in the loss mutualization process. SITG often comes into play twice. First, right after the defaulter-pay resources. The second layer of SITG is often used after the prefunded DF assets of surviving members are depleted. That is, the second layer of SITG can be used before surviving members' unfunded DF.[5] The first and second layer of SITG are denoted by S and $\tilde{S}$ in this paper (Section 2).

This paper also analyzes what we call the *monolayer* default waterfall where the IM pool is used for loss mutualization as a separate layer of DF does not exist in addition to IM in some CCPs (Section 6). This case is becoming increasingly important as some of the systemically importance securities CCPs in the U.S. operate under this structure. In a recent contract-theoretic work, Kuong and Maurin (2023) show that the CCP default waterfalls in their most general and abstract forms may be optimal when the collateral cost is not too high.[6] Ghamami et al. (2021) and Ghamami (2020) show that collateral in the form of IM may increase contagion and financial stability risks.

The main goal of this paper is to introduce a robust framework for the design of SITG. As shown in recent CCP surveys (Thiruchelvam (2022); Walker (2023)),

- SITG is often a very small fraction of member prefunded resources. For instance, SITG represents 1 percent of the default fund at the UK's largest CCP, the London Clearing House (LCH) for interest rate swaps;

- SITG levels vary widely across CCPs; and

One recommendation was: "*requiring CCPs to make material contributions of their own capital to the default waterfall in two separate tranches.*"

[5]When losses cannot be covered by prefunded financial resources, CCPs can often ask the surviving members to make additional contributions to the default fund. These are referred to as unfunded DF contributions.

[6]See Proposition 3 and Corollary 1 in Kuong and Maurin (2023).

- policymakers do not have a quantitative methodology for evaluating the sufficiency of SITG levels (Murphy, 2017).

The goal of this paper is to address these shortcomings. Unlike bank regulation, CCP regulation is mostly principles-based (Ghamami, 2015). Capital regulation of CCPs may not correspond to their risk profiles. Our proposed formulations of SITG can be viewed as risk-based lower bounds on minimum CCP capital requirements (Section 5).[7]

Conflicts of interest and agency problems are ubiquitous in OTC markets. They arise in different forms in centrally cleared markets. When left unmitigated, the ones with links to CCP risk management may have adverse financial stability consequences. CCPs can be viewed as counterparty credit risk insurance providers. The classical moral hazard problem here is that clearing members may be incentivized to take more counterparty credit risk. A well-designed loss mutualization scheme and adequate collateral requirements could mitigate this moral hazard problem. In the mechanism design approach of Biais et al. (2016) and Bolton and Oehmke (2015), adequate levels of collateral (margin) can mitigate moral hazard problems of this type in derivatives markets.

Given that default losses can be mutualized among surviving members, in the absence of adequate levels of SITG, CCPs may not be incentivized to properly monitor couterparty credit risk. CCP risk management practices could subsequently become questionable. A well-designed SITG can mitigate this variation of moral hazard. This agency problem can become subtle at member-owned CCPs. Unlike investor-owned CCPs, one may think that managers under a *members' cooperative* ownership structure would be naturally incentivized to put in place robust risk management frameworks. This need not be case as we argue by drawing on the work of Hart and Moore (1996) and Hansmann (2013). A member-owned CCP could face collective decision-making complications that may ultimately lead to insufficient levels of SITG. We show that this problem can be exacerbated under CCPs with heterogeneous membership (Sections 2 and 3). Membership is not homogeneous at large clearinghouses. Our results can be contrasted with the contract-theoretic work of Huang (2019) where a member-owned CCP is modeled as a welfare-maximizing social planner (public utility).

Taking the default waterfall as given, we develop an economic framework to analyze central clearing *risk management agency problems* and design SITG to mitigate them. We show that conditional on a member's default, when $S = 0$, surviving member DF assets are more exposed to losses compared to losses that the CCP could face under member prefunded resources. Quantifying the corresponding loss probabilities, we introduce incentive compatibility constraints (ICCs) and formulate SITG to mitigate

[7] The survey study by Menkveld and Vuillemey (2021) highlights the fact that there is little work on CCP regulation. One of the goals of this paper is to fill this gap.

the risk management moral hazard problems. In our setting, S, in its simplest form, is formulated as a percentage of the total DF, which is denoted by D in this paper.

Consider the CCP's tail exposure to each member conditional on its default. Ordering these (tail) loss exposure estimates, for simplicity, assume that member one creates the largest exposure.[8] We call the ratio of the CCP's largest exposure to its aggregate exposures the *concentration ratio* and denote it by c_1. We show that, in its simplest form, when

$$S = (1 - c_1)D,$$

some of the ICCs are satisfied, and the CCP-member risk management incentives can become more aligned (Section 3.2). Our results can be contrasted with Kuong and Maurin (2023) who present a rationale for decreasing the SITG level as the number of members increases. In our setting, when the number of members increases, the concentration ratio decreases so $S = (1 - c_1)D$ increases.

We also show that when $\tilde{S} = 0$, member unfunded DF contributions are more exposed to losses compared to the CCP loss exposures. This can distort risk management incentives. The moral hazard problem can be mitigated by formulating $\tilde{S}$ that satisfies a set of ICCs (Section 3.3). Similar to our formulation of S, we show that $\tilde{S}$ is formulated as a percentage of D. We use a modern approach to extreme value theory (EVT) to design SITG in its most robust form (Section 4). We argue that it is natural to use the *threshold exceedances* approach (Embrechts et al. (1997); McNeil et al. (2015); Tsay (2010)) to model the conditional distribution of (default) losses in excess of initial margin by the Pareto distribution (Assumption 4.1).

In our framework, the total SITG, $S + \tilde{S}$, can be expressed as a fraction of the total default fund size. Our numerical studies in Section 5.1 indicate that for realistic parameters, this leads to SITG levels above 15-20 percent of the total default fund size. This in turn leads to estimates of lower bound for CCP equity capital in terms of total DF.

Our findings have implications for the adequacy of bank capital requirements for exposure to CCPs. The Basel Committee on Banking Supervision (BCBS) has developed these *CCP risk capital* rules (BCBS, 2023). Example 5.3 shows that CCP risk capital rules can be improved as central clearing risks may be underestimated in the current regulatory regime.

The largest securities clearinghouses in the U.S. operate under the monolayer default waterfall. We show that monolayer CCPs may need to hold significantly higher levels of SITG to mitigate risk management agency problems (Section 6). We can approximate the ratio of the monolayer CCP SITG to the multilayered CCP SITG under similar ICCs. This ratio can be roughly

[8] We often refer to this member (member 1) as the *largest member*, or the member to which the CCP has the largest exposure.

equal to the ratio of total IM to total DF under the multilayered default waterfall. In practice, IM can be 10 times or more larger than DF (Ghamami and Glasserman, 2017). According to the recent *Risk.net* articles, monolayer CCPs' capital contribution to the default waterfall is less than 1 percent of member prefunded resources. Our framework indicates that higher levels of SITG may be required to mitigate potential risk management incentive distortions.

The rest of this paper is organized as follows. Section 2 reviews the typical multilayered default waterfall. It shows that CCPs may not be incentivized to allocate their own capital to the default waterfall. Section 3 develops our basic framework. It captures risk management agency problems discussed earlier and shows that they can be mitigated by holding adequate levels of SITG. Section 4 models the tail of the loss distributions with the Pareto distribution, it then develops our robust framework for formulating S and $\tilde{S}$. Section 5 introduces a lower bound for CCP regulatory capital, it also tests the adequacy of CCP risk capital rules. Section 6 analyzes the monolayer default waterfall and compares it with the multilayered waterfall. In Section 7, we discuss additional implications of our investigation.

2 Default Waterfall and CCP Equity Capital

After providing a brief overview of the default waterfall, we argue that in the absence of governmental regulation, CCPs may not be incentivized to make adequate equity capital contributions to the default waterfall. We also note that SITG could be viewed as minimum CCP regulatory capital requirements.

2.1 Default Waterfall

Consider a CCP that clears transactions in an asset class for N clearing members indexed by $i = 1, ..., N$. We denote by U_i the exposure of the CCP to member i over a given *risk horizon*, which is often referred to as margin period of risk (MPOR). U_i is a positive random variable that in part captures member i's portfolio value changes over the MPOR.

Each member i with open positions contributes an initial margin M_i to the CCP. At multilayered CCPs, IM posted by each member may only be used to absorb losses arising from the member's portfolio, but cannot be used to offset losses of other members or other losses incurred by the CCP. We discuss CCPs under the monolayer waterfall structure in Section 6. Regulatory guidelines require IM to cover the exposure with a certain confidence level, typically with a minimum of 99%.[9] We represent M_i as a quantile of U_i for some confidence level $1 - q$ where $q \leq 0.01$.

[9]See Principle 6 in BIS (2012)

The (residual) exposure net IM to member i is thus given by $(U_i - M_i)^+ = \max(U_i - M_i, 0)$. The magnitude of the CCP's exposure to member i net IM in *extreme but plausible scenarios* is often modeled using a risk measure ρ associated with the random variable $(U_i - M_i)^+$ at confidence level $1 - q_D$,

$$E_i = \rho_{q_D}\Big((U_i - M_i)^+\Big), \tag{1}$$

with $q_D < q$, we note that ρ can be Value-at-Risk (VaR), Expected Shortfall (ES), Range VaR or any other loss-based risk measure (Cont et al., 2013). In this paper, we use VaR unless it is mentioned otherwise,[10]

$$\rho_{q_D}\Big((U_i - M_i)^+\Big) = VaR_{q_D}((U_i - M_i)^+).$$

Each member also contributes to the CCP's prefunded DF. D_i represents the contribution of member i to DF, and D denotes

$$D = \sum_{i=1}^{N} D_i$$

the size of the total DF. Regulatory guidelines require that DF covers potential losses incurred due to a given number of member defaults – at least one and often two for systemically important CCPs.[11] Denoting by $E^{(i)}$ the ith the largest exposure,

$$E^{(1)} = \max\left(E_i, \quad i = 1..N\right) \geq E^{(2)} \geq ... \geq E^{(N)} = \min\left(E_i, \quad i = 1..N\right),$$

the *cover-one*-based DF leads to a prefunded default fund given by the size of the CCP's largest tail exposure,

$$D = \max\left(E_i, \quad i = 1..N\right) = E^{(1)}. \tag{2}$$

The cover-two-based DF is intended to cover the simultaneous default of two members that would jointly create the CCP's largest (tail) exposure. The cover-two DF can be formulated as[12]

$$D = E^{(1)} + E^{(2)}. \tag{3}$$

To simplify the exposition, the analysis in the main body of the paper mostly

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[10]This is to simplify the exposition and focus on the main results. It is not difficult to carry out the analysis when ρ is taken as Expected Shortfall.

[11]See Principle 4 in BIS (2012). Systemically important securities CCPs in the U.S. operate under the cover-one DF rule.

[12]It can also be formulated as $\max\Big\{\rho_{q_D}\Big((U_i - M_i)^+ + (U_j - M_j)^+\Big); i \neq j,\ i, j = 1..N\Big\}$. As long as the subadditivity property holds, a cover-two DF formulated as (3) leads to a more conservative DF.

focuses on the cover-one DF. Section 4.5 and Appendix A.5 extend our analysis and results to the more general setting, i.e., cover-n DF; $2 \leq n \leq N$. Unlike IM which has become standardized to some extent after the GFC at derivatives CCPs, the modeling and sizing of DF and its allocation to members varies considerably across CCPs, (Cont (2015); Ghamami (2015); Ghamami and Glasserman (2017)). Some derivatives CCPs allocate DF to members proportional to tail exposures,

$$D_i = D \quad \frac{E_i}{\sum_{j=1}^{N} E_j}. \tag{4}$$

This seems plausible intuitively as the overall size of DF depends on the magnitude of these exposures. However, other allocation schemes also exist. For instance, some CCPs allocate the total DF proportional to initial margin, trading volume, open interest, or a weighted mixture of all these quantities.

The order in which the default waterfall financial resources are used to absorb losses when a member defaults can be summarized as follows.

- The first layer of protection against losses is provided by IM posted by the defaulting member.
- If the loss exceeds the IM contribution of the defaulting member, it's prefunded default fund contribution is used to cover any additional losses. If the loss exceeds the sum of the defaulting member's IM and DF contribution,
- The CCP makes a (capped) contribution to offset the remaining loss, this contribution is often referred to as SITG. We denote the size of this first layer of SITG by S.
- The default fund contributions of surviving members are used to absorb the potential remaining losses. These losses can be mutualized and allocated across members proportional to their contribution D_i to the default fund.
- Once the prefunded default funds are exhausted, the CCP may use various *recovery* mechanisms to restore its funding resources (Cont, 2015; Duffie, 2015). These typically include: (i) an additional capital contribution by the CCP – we denote the size of this second layer of SITG by $\tilde{S}$; (ii) additional DF contributions (*assessments*) by surviving members, capped at the level of their prefunded default fund contribution; and (iii) other recovery measures, such as variation margin haircuts.

2.2 CCP Capital Contribution to the Default Waterfall

In the absence of SITG regulation, investor-owned CCPs may not be incentivized to make capital contributions to the default waterfall.[13] This can be shown in different ways. In what follows, we first illustrate this in a very simple and stylized way. Conditional on the default of member j, the CCP's loss up to this stage of the default waterfall can be written as

$$L = \min\left\{(U_j - M_j - D_j)^+, S\right\} + \min\left\{(U_j - M_j - D - S)^+, \tilde{S}\right\}. \quad (5)$$

CCP revenue is proportional to the volume of cleared transactions. Consider an investor-owned CCP. Let V and ϕ denote the CCP's average clearing volume over a given period of time and the clearing fee. The CCP's expected net profit could then be approximated by

$$\phi V - E[L]. \quad (6)$$

Suppose that the CCP maximizes expected net profits by choosing optimal levels of S and $\tilde{S}$. In the absence of capital constraints, the CCP solves this problem by setting $S = 0$ and $\tilde{S} = 0$. That the CCP chooses zero capital contribution is clear from the above formulation.

If regulators require clearinghouses to contribute minimum regulatory capital to the default waterfall, CCPs may then be incentivized to adjust their capital structure and improve their risk management frameworks to maximize (6) given the minimum regulatory-enforced $S > 0$ and $\tilde{S} > 0$. For instance, suppose that V is a decreasing function of IM. That is, all else equal, margin levels above a regulatory minimum reduce the volume of trades that the CCP can attract. Then, given regulatory-driven $S > 0$ and $\tilde{S} > 0$, the profit maximization problem would be solved by, for instance, choosing optimal levels of IM, S, and $\tilde{S}$ above the regulatory minimum.

CCP Objective Function

The approximate net profit formulation (6) abstracts away from the total level of CCP capital, costs associated with it, and the cost of a CCP failure. We now sketch the augmented objective function of an investor-owned CCP in the presence of these costs.

Consider the case of a single representative CCP. Given the typical multilayered default waterfall, suppose that $E_t = S + \tilde{S} + E_s$ is the total capital of the investor-owned CCP, where E_s is the portion of CCP capital that is

[13]We consider two broad classes of ownership structure: (i) the most common one, which is outside ownership; and (ii) CCPs that can be viewed as members' cooperatives. These are referred to as investor-owned and member-owned (user-owned) CCPs in this paper. It is insightful to view them as capital and consumer cooperatives (Hansmann, 2013).

not allocated to the default waterfall. We also assume that the unfunded DF is capped by a multiple of prefunded DF, βD, with $\beta > 0$. Conditional on the default of member j, consider the loss to the CCP in excess of member resources and the CCP's total capital,

$$L_e = (U_j - M_j - D - \beta(D - D_j) - E_t)^+. \tag{7}$$

In the presence of $E_t > 0$ and conditional on the default of member j, the private profit-seeking objective of an investor-owned CCP would be to maximize

$$\phi V - E[L] - E[L_e] - c(E_t) - c_p Q(S, \tilde{S}, E_s), \tag{8}$$

where the first two terms come from (6), $c(E_t)$ is the social cost of CCP capital, with $c(\cdot)$ being an increasing, convex function, c_p is the private cost of a CCP failure, and $Q(S, \tilde{S}, E_s)$ is the probability of such a failure, with $Q(\cdot)$ being a multivariate decreasing, convex function. A basic and standard assumption in this paper is that CCP failures are costly for society. we assume that, in the absence of capital requirements, CCPs do not fully internalize the costs of their own failures. That is, the social cost of a CCP failure, c_s, is larger than the private cost c_p. The second basic and standard assumption is that there is a social cost associated with having more CCP capital, and the capital structure irrelevance principle (Modigliani and Miller, 1958) fails for CCPs.[14]

Now, suppose that E_t is given, for instance, it is set by regulators but the allocation of it to S, $\tilde{S}$, and E_s is left to the CCP. Then, it is not difficult to see that the CCP maximizes its objective by setting $S = \tilde{S} = 0$. That is, it does not allocate its own equity capital to the default waterfall. While the augmented objective function (8) is more comprehensive than the stylized net profit formulation (6), to see that investor-owned CCPs may not be incentivized to have any SITG, it suffices to focus on the simpler formulation (6). In the absence of SITG regulation, policymakers may underestimate the probability of CCP failure and so the corresponding social costs of such a failure. In the next sections, we show that when $S = \tilde{S} = 0$, CCP risk management incentives can be distorted. This suggests that adequate

[14]Greenwood et al. (2017) use similar assumptions to formulate the cost of capital and bank failure in their setting. As in Greenwood et al. (2017), the only cost of equity that is incorporated in the objective function (8) is the one associated with the stock of equity on the balance sheet. Under the simplifying assumption that the private costs of equity finance equal social costs, Greenwood et al. (2017) show that bank risk-based capital regulation can be optimal (in the steady state). We do not need to deviate from this simplifying assumption to show that in the absence of SITG regulation, CCP equity capital levels can be socially sub-optimal.

levels of STIG can improve CCP risk management and subsequently decrease the CCP's default probability. The following example may be insightful. Suppose that the *true default probability* of the CCP is represented by the logistic distribution function

$$Q(S, \tilde{S}, E_s) = \frac{\exp(\zeta_0 + \zeta_1 S + \zeta_2 \tilde{S} + \zeta_3 E_s)}{1 + \exp(\zeta_0 + \zeta_1 S + \zeta_2 \tilde{S} + \zeta_3 E_s)}, \tag{9}$$

where $\zeta_i < 0$, with $i = 0, 1, 2, 3$. In the absence of SITG regulation, the regulator (social planner) may obtain sub-optimal levels of CCP equity capital by maximizing the following objective function

$$\phi V - E[L] - E[L_e] - c(E_t) - c_s \hat{Q}(E_t), \tag{10}$$

where instead of the CCP's true default probability function (9), the following inaccurate estimate of it is used,

$$\hat{Q}(E_t) = \frac{\exp(\eta_0 + \eta_1 E_t)}{1 + \exp(\eta_0 + \eta_1 E_t)}, \tag{11}$$

where $\eta_0, \eta_1 < 0$.

Outside Ownership versus Members' Cooperatives

It is well-known that some of the clearinghouses are member-owned (user-owned), for instance, the Options Clearing Corporation (OCC) and subsidiaries of the Depository Trust and Clearing Corporation (DTCC) in the U.S., and the Japanese Security Clearing Corporation (JSCC). Member-owned CCPs are often treated as public utilities or welfare maximizing social planners in the existing contract-theoretic or mechanism design models of central clearing.[15] Since member-owned CCPs are not owned by individuals but by other profit-seeking firms, economic analysis of this class of CCPs under the assumption that they are welfare maximizing public utilities may not have fruitful policy applications.

While this paper does not address the governance and ownership structure of CCPs, we believe that it is important to draw on the work of Hart and Moore (1996) and Hansmann (2013). In their analysis of cooperatives, Hart, Moore, and Hansmann (use different approaches to) illustrate the importance of a *control-based view of ownership* and note that effective governance of a members' cooperative could be challenging, and the cost of col-

[15]See, for instance, Section 5 of Huang (2019). More generally, CCPs have sometimes been modeled as public utilities, see, e.g. page 1677 in Biais et al. (2016).

lective decision-making could be rather high under this ownership structure. They show that outside ownership can be more efficient than a members' cooperative when the membership becomes less homogeneous. Membership is not homogeneous by any measures at systemically important CCPs.

Member-owned CCPs in advanced economies are large businesses that are run by managers. Exerting effective control on management can be particularly difficult at a members' cooperative. We argue that SITG should be regulated at member-owned CCPs. Otherwise, risk management agency problems may adversely impact financial stability. In the absence of effective capital regulation, member-owned CCPs may allocate insufficient levels of SITG to the default waterfall due to: (i) collective decision-making complications and membership heterogeneity; and (ii) the fact that CCP equity capital could be more costly than collateral in the form of member default fund contributions. In the next section, we sketch the objective function of member-owned CCPs in comparison with (6) and illustrate that unlike investor-owned CCPs, member-owned CCPs may have the incentives to allocate some levels of SITG to the default waterfall. We also show how membership heterogeneity may lead to insufficient SITG levels.

CCP Capital Regulation

Principles 2, 4, and 15 of the PFMI outline minimum regulatory capital requirements for CCPs and indicate that parts of CCPs' own financial resources should be allocated to the default waterfall (BIS (2012, 2017)). Regulators and CCPs are then expected to specify the form and size of total regulatory capital and the amount that should be allocated to the default waterfall. In Europe, for instance, CCP capital requirements are the sum of four components: (i) capital requirements for winding down or restructuring activities; (ii) capital requirements for operational and legal risk; (iii) capital requirements for credit, counterparty, and market risk;[16] and (iv) capital requirements for business risk, (EUR (2012)). The European Market Infrastructure Regulation (EMIR) then requires CCPs to allocate 25 percent of the total regulatory capital to the default waterfall, (EUR (2013); McLaughlin (2018)). Under our proposed framework, $S + \tilde{S}$, can be viewed as a lower bound for CCP regulatory capital. It can also be directly compared with EMIR's 25 percent rule.

3 Default Losses and Skin in the Game

In this section, we compare default losses from the perspective of the CCP and members and illustrate that the CCP and members are disproportion-

[16]This component of CCP capital requirements in Europe is formulated using BCBS rules for credit, counterparty, and market risk capital requirments.

ately exposed to default losses. The underlying economic argument is that when SITG is designed to lower potential losses to member DF assets, risk management incentive distortions can be mitigated.

3.1 Member Perspective

Consider the exposure of a surviving member to the default of another member. If member j defaults, the potential loss to the DF assets of a non-defaulting member $i \neq j$ is given by

$$\underbrace{(U_j - M_j - D_j - S)^+}_{\text{Loss imputed to the remaining default fund}} \quad \underbrace{\left(\frac{D_i}{D - D_j}\right)}_{\text{Fraction imputed to member } i}. \tag{12}$$

If we limit member i's DF losses to its prefunded default fund contributions, D_i, the resulting exposure of member i due to the default of member j becomes

$$L_i^j = D_i \quad \min\left(\frac{(U_j - M_j - D_j - S)^+}{D - D_j},\ 1\right). \tag{13}$$

Note that the ratio L_i^j/D_i, i.e. the relative loss on DF assets is the same for all non-defaulting members and only depends on the severity of the default event and on the DF allocation rule. Surviving members incur losses if the defaulting member's loss exceeds its IM, its DF, and the first layer of SITG. That is,

$$L_i^j > 0 \quad \Longleftrightarrow \quad U_j > M_j + D_j + S. \tag{14}$$

This result holds regardless of the rules used for sizing the default fund and for its allocation across members.

We now consider the case where DF contributions are allocated proportional to tail exposures net IM as in (4). Given (14), non-defaulting members will incur losses if

$$U_j > M_j + D\frac{E_j}{\sum_{k=1}^N E_k} + S.$$

In the cover-one case where $D = E^{(1)}$, this inequality becomes

$$U_j > M_j + E_j\frac{E^{(1)}}{\sum_{k=1}^N E_k} + S \tag{15}$$

where E_j is defined in (1). The right side of (15) involves what we refer to as the (default fund) concentration ratio

$$c_1 = \frac{E^{(1)}}{\sum_{k=1}^N E_k}, \tag{16}$$

which measures the relative magnitude of the CCP's largest exposure. As will be further discussed later, this ratio can play an important role in the risk analysis of the default waterfall. Since $c_1 < 1$, in the absence of any CCP capital contribution, the probability that non-defaulting members take a loss is *always* larger than q_D,

$$P(U_j > M_j + D\frac{E_j}{\sum_{k=1}^{N} E_k}) \geq P(U_j > M_j + E_j) = q_D.$$

Note that this is the case regardless of the magnitude of the default. In short, setting $S = 0$ gives

$$P(L_i^j > 0) \geq q_D, \tag{17}$$

where $i \neq j$.[17]

3.2 Skin in the Game: First Layer

Conditional on the default of member j, when $S = 0$, the potential loss to the CCP in the presence of IM and prefunded DF is

$$L_0^j = (U_j - M_j - D)^+,$$

we can write

$$P(L_0^j > 0) \leq q_D. \tag{18}$$

To simplify the notation, hereafter, we assume that member 1 (N) is the member to which CCP has the largest (smallest) exposure, i.e. $E^{(1)} = E_1$, ($E^{(N)} = E_N$). Under the cover-one rule, we have

$$P(L_0^1 > 0) = q_D.$$

We note that in the absence of any CCP capital contributions, non-defaulting members are more likely than the CCP to incur default losses

$$P(L_i^j > 0) \geq q_D \geq P(L_0^j > 0). \tag{19}$$

This inequality captures an important conflict of interest between the CCP and members from risk management perspective: in the absence of SITG, (non-defaulting) members are more exposed to default losses than the CCP. This moral hazard problem could be mitigated by lowering loss probabilities associated with non-defaulting member prefunded DF assets. More specif-

[17] If quantified, measured, and monitored appropriately, loss probabilities associated with members' DF contributions can be used to construct credit ratings for CCPs' default fund.

ically, we formulate S in a way that the following incentive compatibility constraint is satisfied

$$P(L_i^j > 0) \leq q_D. \tag{20}$$

To further elaborate on the *incentive compatibility* aspect of this constraint, consider two scenarios. In scenario A, we set

$$S_l = (1 - c_1)D. \tag{21}$$

Let $c_N = E_N / (\sum_{i=1}^{N} E_i)$. In scenario B, we have

$$S_u = (1 - c_N)D. \tag{22}$$

Note that $S_l \leq S_u$ as $c_N \leq c_1$.

Scenario A Conditional on the default of member 1, consider loss probabilities from the perspective of member i and the CCP. Note that given

$$P(L_i^1 > 0) = P(U_1 - M_1 > D_1 + S), \text{ and } P(L_0^1 > 0) = P(U_1 - M_1 > D),$$

setting $S = D - D_1 = (1 - c_1)D$ gives

$$P(L_i^1 > 0) = P(L_0^1 > 0) = q_D. \tag{23}$$

That is, under (21), *large* counterparty default loss probabilities become perfectly aligned from the CCP and member perspectives. Moreover, we will show in the next section that under our EVT-based framework, in this scenario, the loss probability q_D becomes an upper bound on member loss probabilities[18]

$$P(L_i^j > 0) \leq q_D.$$

While CCP and member risk incentives are fully aligned under largest counterparty default losses in scenario A, non-defaulting members are more exposed to *other (remaining) counterparty default losses* compared to the CCP,

$$P(L_i^j > 0) \geq P(L_0^j > 0); \quad j \neq 1, i. \tag{24}$$

[18]In the next section, we argue that the proposed EVT-based framework is the natural one to be considered for the design of SITG. It is intuitive economically and financially to model the tail of (conditional)loss distributions with the Pareto distribution.

We will return to this inequality shortly.

Scenario B When $S_u = (1 - c_N)D$, the following inequality holds

$$P(L_i^j > 0) = P(U_j - M_j > D + D_j - D_N) < q_D, \tag{25}$$

as $D_j \geq D_N$. That is, the overarching ICC (20) is satisfied. Moreover, under (22), we can write

$$P(L_i^j > 0) \leq P(L_0^j > 0), \tag{26}$$

for all $j \neq i$. That is, under S_u, non-defaulting members are all less exposed to counterparty default losses compared to the CCP. In short, $[S_l, S_u]$ provide a range of values for the first layer SITG where the moral hazard problem can be mitigated to different quantifiable degrees. When policymakers aim for regulating the *minimum* equity capital, S_l could be a natural choice for SITG.

Remark That the CCP has small exposure to default losses,

$$P(L_0^j > 0) \leq q_D < q,$$

relies on the assumption that DF has been sized adequately. These loss probabilities need not remain small if DF does not capture client clearing risks properly. For instance, if in estimating DF, the CCP's exposure to member 1 conditional on its default, U_1, does not take into account portfolios that member 1 has cleared through the CCP on behalf of its customers, $P(L_0^1 > 0)$ could exceed q_D and q. Suppose that member 1 defaults and its IM covers losses associated with member 1's *house (proprietary) account.* Over a period of time till *client accounts* can be ported to a non-defaulting member, the CCP may need to make payments to member 1's customers. If DF is not sized properly to cover losses that could arise due to member 1's default (or the default of some of its customers), the resilience of the CCP can be adversely impacted. This would all depend on specifics of client clearing models, an important topic that is not addressed in this paper. The default waterfall should evolve proportionately to the risk profile of the CCP. Increased client clearing should increase IM, DF, and SITG adequately.

3.3 Skin in the Game: Second Layer

The second layer SITG could be viewed as a buffer against potential losses to members' unfunded DF assets. From the perspective of non-defaulting member i and conditional on the default of member j, the total loss to member i's prefunded and unfunded default fund assets can be represented by

$$\tilde{L}_i^j = L_i^j + \left(U_j - M_j - S - D - \tilde{S}\right)^+ \frac{D_i}{D - D_j}. \tag{27}$$

When the unfunded default funds are capped by D_i or a multiple of D_i, denoted by βD_i; $\beta > 0$, the second term on the right side above is replaced with the minimum of it and D_i (βD_i). Given (27), the probability that potential losses to member i exceed its prefunded DF assets is given by

$$P(\tilde{L}_i^j > D_i) = P(U_j - M_j > D + S + \tilde{S}) \tag{28}$$

This is the likelihood that member i's unfunded DF resources would come into play due to the default of member j. This probability is bounded above by q_D when $j \neq 1$, or when S or $\tilde{S}$ are positive. Note that when $S = \tilde{S} = 0$, the probability that the DF of member i is depleted due to the default of the largest member is equal to q_D,

$$P(\tilde{L}_i^1 > D_i) = q_D. \tag{29}$$

Given $S > 0$, we formulate $\tilde{S}$ to lower the likelihood that member losses exceed their prefunded DF contributions. More specifically, consider a *target loss probability (upper bound)* associated with unfunded DF contributions, $\tilde{\pi}$, where $0 < \tilde{\pi} < q_D$. Given $S > 0$, we specify $\tilde{S}$ in a way that the following constraint

$$P(\tilde{L}_i^j > D_i) \leq \tilde{\pi}, \tag{30}$$

is satisfied. In short, under the most basic form of the incentive compatibility framework, S and $\tilde{S}$ can be formulated such that loss probabilities satisfy ICCs (20) and (30). We now demonstrate that (30) is grounded on similar economic arguments used in the previous section. More specifically, taking the CCP's perspective, conditional on the default of member j, when $\tilde{S} = 0$, the potential loss to the CCP in excess of S and all prefunded and unfounded resources is

$$\tilde{L}_0^j = \left(U_j - M_j - D - S - \beta(D - D_j)\right)^+. \tag{31}$$

Given $S > 0$, note that when $\tilde{S} = 0$, we have

$$P(\tilde{L}_i^j > D_i) > P(\tilde{L}_0^j > 0), \tag{32}$$

for any $j \neq i$. This inequality captures another important conflict of interest between the CCP and its members: in the absence of the second layer SITG, member potential losses in excess of their prefunded DF assets could be larger than the comparable potential loss to the CCP. The target loss probability $\tilde{\pi}$ will be chosen such that ICC (30) would mitigate this moral hazard problem. To elaborate more on this second overarching and basic incentive compatibility constraint, it will be insightful to consider two scenarios. First, we set

$$\tilde{S}_l = \beta D(1 - c_1). \tag{33}$$

In scenario B, the second layer SITG is formulated as follows,

$$\tilde{S}_u = \beta D(1 - c_N). \tag{34}$$

Note that $\tilde{S}_l \leq \tilde{S}_u$.

Scenario A Suppose that $\tilde{\pi} = P(\tilde{L}_0^1 > 0)$. Given $S > 0$, setting $\tilde{S}_l = \beta D(1 - c_1)$ results in

$$P(\tilde{L}_i^1 > D_i) = P(\tilde{L}_0^1 > 0) = P\big(U_1 - M_1 > D + S + \beta(D - D_1)\big). \tag{35}$$

In words, under (33), the CCP and members risk management incentives become fully aligned in terms of potential largest counterparty default losses that would exceed the prefunded resources and the first layer SITG. As will be shown in the next section, under our EVT (Pareto)-based framework,

$$P(\tilde{L}_i^j > D_i) \leq P(\tilde{L}_i^1 > D_i), \tag{36}$$

for $j \neq i, 1$. Consequently, under (33), the explicit incentive compatibility constraint (35) along with the above EVT-driven inequality gives (30). However, we note that in scenario A, we have

$$P(\tilde{L}_i^j > D_i) \geq P(\tilde{L}_0^j > 0), \tag{37}$$

for any $j \neq i$. In words, while $\tilde{S}_l = \beta D(1 - c_1)$ mitigates the moral hazard problem associated with unfunded DF asset losses to some extent, members remain more exposed than the CCP to counterparty default losses that would exceed prefunded resources and the first layer SITG. We will return to inequality (37) shortly.

Scenario B Suppose that $S > 0$ is given. Since $c_N \leq c_1$, setting $\tilde{S}_u = \beta D(1 - c_N)$ gives

$$P(\tilde{L}_i^j > D_i) \leq P(\tilde{L}_0^j > 0), \tag{38}$$

for any $j \neq i$. It is useful to note that under (34), we have

$$P(\tilde{L}_i^N > D_i) = P(\tilde{L}_0^N > 0) = P\big(U_N - M_N > D + S + \beta(D - D_N)\big).$$

That is, under the more restrictive second layer SITG (34), members become less likely than the CCP to incur counterparty default losses that would exceed their unfunded resources and the first layer SITG. Here the target loss probability upper bound associated with ICC (30) could continue to be viewed as $\tilde{\pi} = P(\tilde{L}_0^1 > 0)$. That is, in this scenario, both (30) and the more explicit and restrictive ICC (38) are satisfied. In sum, the second layer SITG that belongs to the range $[\tilde{S}_l, \tilde{S}_u]$ can mitigate this variation of the moral hazard problem linked to CCP and members tail risk management incentives to different quantifiable degrees. Given the regulatory focus on minimum equity capital requirements, policymakers could adopt and appropriately calibrate $\tilde{S}_l$ as an economically sound choice for the second layer of SITG.

3.4 Member-Owned CCPs

Exerting control on managers could be difficult at member-owned CCPs in part due to collective-decision making complications. This agency problem can be exacerbated under membership heterogeneity. Member-owned clearinghouses may have the incentive to allocate some levels of equity capital to the default waterfall. However, unregulated SITG levels may be insufficient and may in turn adversely impact risk management incentives.

Recall the expected net profit formulation (6) at investor-owned CCPs. We now introduce a variation of it that corresponds to member-owned CCPs. Suppose that $\psi_i V_i$ represents member i's gross profit from its trades in a volume of V_i that have been cleared through the CCP. For simplicity, we assume that $V = V_1 + V_2 + ... + V_N$.[19] For simplicity, suppose that all members receive an equal share of the CCP's profit. Then, conditional on the default of member j, member i's expected net profit can be written as

$$\frac{\phi V - E[L]}{N-1} + \big(\psi_i V_i - E[\tilde{L}_i^j]\big),$$

where the second term inside the parentheses can be viewed as member i's *consumer surplus* in the sense of Hart and Moore (1996).[20] Note that $\tilde{L}_i^j$ is

[19]Note that $\sum_{i=1}^N \psi_i$ need not be equal to ϕ as the fee structure at the CCP can be different from each member's profit generating schemes from its trading activities.

[20]In any market, total surplus can be viewed as the sum of total producer surplus and

the total loss to member i's prefunded and unfunded DF assets defined in (27). Taking the typical multilayered default waterfall as given, member i maximizes expected net profit by choosing optimal levels of S and $\tilde{S}$,

$$\frac{\phi V}{N-1} + \psi_i V_i - \left(\frac{E[L]}{N-1} + E[\tilde{L}_i^j] \right). \tag{39}$$

Consequently, the expected net profit of the member-owned CCP conditional on the default of member j becomes,

$$(\phi + \sum_{i \neq j} \psi_i) V - \left(E[L] + \sum_{i \neq j} E[\tilde{L}_i^j] \right). \tag{40}$$

These simple formulations highlight the basic fact that unlike investor-owned CCPs, member-owned CCPs may not be incentivized to set $S = \tilde{S} = 0$.[21] To see this, consider expected losses in (39), and note that CCP managers' expected loss $E[L]$ can be viewed as an increasing function of S and $\tilde{S}$ while member i's expected loss $E[\tilde{L}_i^j]$ can be viewed as a decreasing function of S and $\tilde{S}$. So, an optimal first and second layer SITG could be positive.[22]

Member and CCP expected net profit functions highlight the adverse impact of membership heterogeneity on SITG levels. To see this, consider member expected net profit function (39). It then suffices to note that $\tilde{L}_i^j$ and so $E[\tilde{L}_i^j]$ can be viewed as increasing functions of D_i. In words, since *larger members* contribute more to the DF, their optimal levels of SITG can be larger than that of *smaller members*. Larger members would vote for higher levels of SITG while smaller members would vote for lower SITG levels. In a heterogeneous member-owned CCP, reaching a consensus on an optimal level of SITG can be particularly challenging as members with different levels of DF assets would vote for different levels of SITG.[23]

While conflicts of interest and agency problems in investor-owned and member-owned CCPs are not identical, the outcome could be similar: clearinghouses with socially sub-optimal levels of SITG and CCP capital. This is particularly the case for systemically important CCPs that are also exposed to the too-big-to-fail problem.

We note that our SITG design framework directly applies to member-

total consumer surplus. See Section VII in Hart and Moore (1996).

[21] We can also easily see from (40) that modeling member-owned CCPs as welfare maximizing social planners may not prove useful.

[22] The empirical results of Huang (2019) confirm that member-owned CCPs hold higher levels of SITG compared to investor-owned CCPs, (see Figure 6 of Huang (2019), which uses 2015 quantitative disclosure data from CCPs).

[23] Note that control in the form of voting rights may be allocated according to a simple one-member-one-vote rule.

owned CCPs. For instance, consider inequality (19) that highlights the conflict of interest between the investor-owned CCP and its members from the risk management perspective. At the member-owned CCP, similarly, when $S = 0$, members are more exposed to default losses than the CCP,

$$P(L_i^j > 0) \geq q_D \geq P(L_0^j > 0),$$

and this could disincentivize managers to monitor and manage concentrated risks at clearinghouse adequately. Collective decisions-making problems and membership heterogeneity in the absence of governmental SITG regulation may lead to insufficient levels of SITG. Formulating S such that ICC (20) or (26) are satisfied could mitigate risk management incentive distortions and may also improve the collective decision-making process under this ownership structure.

Remark While the net profit formulations (39) and (40) abstract away from the costs of CCP equity capital and CCP default, our results remain the same when these costs are incorporated into augmented objective functions. Recall the objective function of an investor-owned CCP (8). Given (39) under a members' cooperative ownership structure, the augmented objective function of member i conditional on the default of member j can be approximated by

$$\frac{\phi V}{N-1} + \psi_i V_i - \left(\frac{E[L] + E[L_e] + c^m(E_t) + c_p^m Q^m(S, \tilde{S}, E_s)}{N-1} + E[\tilde{L}_i^j] \right), \quad (41)$$

where L_e is defined in (7), $c^m(E_t)$ is the social cost of the CCP capital under a members' cooperative ownership structure, with $c^m(\cdot)$ being an increasing, convex function, c_p^m is the private cost of the CCP's default, and $Q^m(S, \tilde{S}, E_s)$ is CCP default probability, with $Q^m(\cdot)$ being a multivariate decreasing, convex function. Suppose that E_t is set by the regulators, and that given E_t, member i maximizes (41) by allocating the total capital to S, $\tilde{S}$, and E_s. As discussed earlier, since $E[\tilde{L}_i^j]$ is a decreasing function of S and $\tilde{S}$, the optimal SITG level from the perspective of member i could be positive. However, SITG and total CCP capital levels may be socially sub-optimal in the absence of governmental SITG regulation because: (i) CCP and members do not fully internalize the cost of CCP failure; and (ii) under membership heterogeneity, different members could arrive at different (privately) optimal SITG levels, and this could lead to socially sub-optimal SITG levels at the CCP. Our analysis suggests that SITG regulation may be required to effectively address risk management agency problems under this ownership structure.

4 Skin in the Game: Modeling Tail Risk

We use a modern approach to extreme value theory to model the tail risk associated with default losses. This section introduces our framework in its most robust and general form. The proposed SITG formulations can mitigate risk management agency problems to different measurable degrees.

4.1 Modeling Tail Risk

Default losses often arise during extreme and distressed market conditions. To analyze the distribution of these losses, it is natural to model the *tail* of the loss distributions associated with CCP-member portfolios. A flexible and powerful semi-parametric approach for modeling these distribution tails is to use the generalized Pareto distribution (GPD), (McNeil et al., 2015, Ch.7, Theorem 7.20). It is well-known that GPD becomes an ordinary Pareto distribution in the heavily-tailed scenario, the case used in our study.

More specifically, we use the *threshold exceedances* approach to model the tail of the loss distribution.[24] We represent the conditional distribution of losses in excess of a high threshold as a Pareto distribution, whose tail exponent or shape parameter $\alpha > 1$ quantifies the heaviness of the tail. The natural threshold in our setting is initial margin. We thus assume that default exposures in excess of IM have a Pareto or power-law distribution. Specifically, the following assumption is used to derive our S and $\tilde{S}$ formulations.[25]

Assumption 4.1 (Pareto tail) *The CCP's exposure to member i conditional on its default, U_i, satisfy*

$$P(U_i - M_i > x |\ U_i \geq M_i) = \left(\frac{\kappa_i + x}{\kappa_i}\right)^{-\alpha} = 1 - F(x; \kappa_i, \alpha), \tag{42}$$

$$\text{where} \qquad M_i = VaR_q(U_i) \quad \text{and} \qquad F(x;\kappa,\alpha) = 1 - \left(\frac{\kappa + x}{\kappa}\right)^{-\alpha}$$

is the Pareto distribution with the tail exponent (shape parameter) $\alpha > 1$ and scale parameter $\kappa > 0$.[26]

It is important to note that we are not assuming a parametric form for the entire loss distribution but only for tail events with probability less than q, i.e. for losses beyond IM. This assumption is consistent with risk models

[24] See also Embrechts et al. (1997) and Tsay (2010). The threshold exceedances approach is also referred to as *peaks over thresholds* (POT).

[25] Theorem 7.20 in McNeil et al. (2015) illustrates that: "*GPD is the canonical distribution for modeling excess losses over high thresholds.*" (See page 278 of McNeil et al. (2015)).

[26] As discussed earlier, it is often the case that $q \leq .01$ in risk management applications.

used by more advanced and sophisticated CCPs and is satisfied with high accuracy for many heavy-tailed distributions, such as Student-t, whose tails behave as (42) for high thresholds. Heavier tails correspond to lower values of the tail exponent α. Empirical studies indicate that this assumption is plausible for different asset classes, for instance, for equity and credit portfolios, with α in the range $2-4$ for equity (Cont, 2001) and credit default swap (CDS) portfolios (Cont and Kan, 2011). Higher values of the tail exponent correspond to equity and CDS indices. Recall that

$$E_i = \mathrm{VaR}_{q_D}\Big((U_i - M_i)^+\Big),$$

where $q_D < q \leq .01$. Assuming that DF is allocated according to E_i as in (4), we can write

$$P(U_i - M_i > E_i) = q_D = qP(U_i - M_i > E_i | U_i > M_i) = q\left(\frac{\kappa_i + E_i}{\kappa_i}\right)^{-\alpha},$$

where the last equality follows from Assumption 4.1. This gives

$$\kappa_i = \frac{VaR_{q_D}((U_i - M_i)^+)}{(q_D/q)^{-1/\alpha} - 1}. \tag{43}$$

This expression for κ_i shows that the scale parameter (κ_i) is proportional to the magnitude of losses in excess of IM. In what follows, we assume that default loss distributions satisfy Assumption 4.1 with some tail exponent α and a scale parameter κ_i that may vary across members. Note that (43) implies

$$\frac{E_i}{\sum_{j=1}^N E_j} = \frac{\kappa_i}{\sum_{j=1}^N \kappa_j}, \tag{44}$$

for any $0 < q_D < 1$. We represent this ratio by c_i. We will shortly return to concentration ratio c_1.

Remark Let σ_i^2 denote the variance of U_i. Following Lemma A.2, if we assume that $U_i/\sigma_i \sim T(0, \nu)$ has a mean-zero Student-t distribution with $\nu > 1$ degrees of freedom, we will have

$$\frac{M_i}{\sum_{j=1}^N M_j} = \frac{E_i}{\sum_{j=1}^N E_j} = \frac{\kappa_i}{\sum_{j=1}^N \kappa_j}.$$

That is, when default exposure distributions over MPOR are modeled by Student-t distributions, total DF can be allocated equivalently to members using E_i or M_i. Concentration ratio can also be approximated based on IM. We will revisit this assumption in Section 6.

4.2 Skin in the Game: First Layer

We continue to assume that the CCP has its largest (smallest) exposure to member 1 (N). The following proposition gives the conditional probability distribution of the largest counterparty default loss L_i^1 to member $i \neq 1$.

Proposition 4.2 *Under Assumption 4.1, the probability distribution function of member i's loss conditional on the default of the member to which the CCP has the largest exposure is given by,*

$$P(L_i^1 > x) = q\left[1 + \left((\frac{q}{q_D})^{1/\alpha} - 1\right)\left(c_1 + \frac{S}{D} + \frac{(1-c_1)}{c_i}\frac{x}{D}\right)\right]^{-\alpha} \tag{45}$$

As illustrated earlier, in the absence of any distributional assumptions, when $S = 0$, we have $P(L_i^1 > 0) \geq q_D$ and $P(\tilde{L}_i^1 > D_i) = q_D$, which can be easily confirmed in the Pareto-based formulation. Working backwards, we can compute S which corresponds to a given target loss probability $\pi > 0$. For instance, suppose that $\pi = P(L_i^1 > 0)$. Solving for S yields,

$$P(L_i^1 > 0) = \pi \iff S = \left(\frac{(\frac{q}{\pi})^{1/\alpha} - 1}{(\frac{q}{q_D})^{1/\alpha} - 1} - c_1\right) D. \tag{46}$$

Our basic overarching objective in formulating S is to lower loss probabilities associated with member DF assets in a way to achieve ICC (20). Setting $\pi \leq q_D$ satisfies this criterion as will be shown shortly. Before doing so, we revisit scenario A in the previous section, where *aligning* member and CCP *largest* counterparty default loss probabilities is achieved by choosing S such that π becomes equal to $P(L_0^1 > 0) = q_D$. Choosing $\pi = q_D$ simplifies the above equation to (21)

$$S_l = (1 - c_1)D,$$

which is the formulation we derived earlier without making any distributional assumptions. When S is sized according to (21), in the event of the default of the largest member, the probability that any surviving member incurs any losses will be q_D. In short, setting $S = (1 - c_1)D$ gives (23).

Concentration Ratio and the Default Fund

It is useful to highlight the nonlinear dependence of $S_l = (1 - c_1)D$ on DF and subsequent implications for the concentration ratio. Suppose that $q_D \in (0, q)$ is given, viewing S_l as a function of $E_1 = \mathrm{VaR}_{q_D}((U_1 - M_1)^+)$,

$$S_l = \left(1 - \frac{E_1}{\sum_{i=1}^N E_i}\right) E_1,$$

it is straightforward to see that $E_1 = E/2$ (with $E = \sum_{i=1}^{N} E_i$), maximizes S_l. That is, a CCP with the concentration ratio of .5 would need to have the maximum level of $S_l = E/4$ to satisfy ICCs (20) and (23). As the concentration ratio increases, the size of DF increases, and so SITG will be sized based on lower percentages of DF. Lower levels of c_1 lead to smaller default funds, and so SITG will be specified based on higher portions of DF. For instance, $c_1 = .8$ gives $S_l = .2D$, while $c_1 = .2$ gives $S_l = .8D$. Now, suppose that c_1 is fixed and view S_l as a function of D. Note that with a given c_1, DF is a decreasing function of q_D. Lower q_D lead to larger default funds, and as D increases, the size of S_l will increase to ensure that member default loss probabilities are aligned with those of the CCP – that is, to ensure that ICCs (20) and (23) are satisfied.

4.3 Exposure to Other Defaults

Proposition 4.3 gives the probability distribution function of L_i^j, the loss to surviving member i's DF assets conditional on the default of member $j \neq 1$.

Proposition 4.3 *Under Assumption 4.1, the probability distribution function of member i's loss conditional on the default of $j \neq i$ is given by*

$$P(L_i^j > x) = q\left[1 + \left((\frac{q}{q_D})^{1/\alpha} - 1\right)\left(c_1 + \frac{S}{E_j} + \frac{x}{E_j}\frac{(1-c_j)}{c_i}\right)\right]^{-\alpha}. \quad (47)$$

It follows from Propositions 4.2 and 4.3 that the highest level of tail risk corresponds to the default of member 1 with the highest stress loss over margin

$$P(L_i^j > x) \leq P(L_i^1 > x),$$

for any $j \neq i, 1$. That is, the largest DF exposure of members also corresponds to the largest exposure of the CCP. By contrast, if $S = 0$, under Proposition 4.3, all members have the same loss probability regardless of the defaulter

$$P(L_i^j > 0) = P(L_i^1 > 0) \geq q_D.$$

Using Propositions 4.2 and 4.3, it is not difficult to see that setting $S_l = (1 - c_1)D$ results in the following bound for member loss probabilities,

$$P(L_0^j > 0) \leq P(L_i^j > 0) \leq q_D.$$

This shows that our formulation of S in scenario A lowers loss probabilities associated with member DF contributions and also aligns member and CCP large loss probabilities in that (20) and (23) are guaranteed to hold.

Now, consider SITG in scenario B. Recall that under $S_u = D(1 - c_N)$, ICCs (20) and (26) are satisfied,

$$P(L_i^j > 0) \le P(L_0^j > 0) \le q_D.$$

It is straightforward to use Proposition 4.3 to quantify the difference between loss probabilities under S_l and S_u.[27]

4.4 Skin in the Game: Second Layer

In the design and analysis of $\tilde{S}$, we use Proposition 4.4, which gives our Pareto-based formulation of member and CCP loss probabilities ((28) and (32)) discussed in the previous section.[28]

Proposition 4.4 *Under Assumption 4.1, and given $S, \tilde{S} > 0$, the probability that member i's loss conditional on the default of $j \neq i$ exceeds its prefunded DF resources is given by*

$$P(\tilde{L}_i^j > D_i) = q\left[1 + \left((\frac{q}{q_D})^{1/\alpha} - 1\right)\left(\frac{D}{E_j} + \frac{S}{E_j} + \frac{\tilde{S}}{E_j}\right)\right]^{-\alpha}. \tag{48}$$

Also, conditional on the default of member j and in the absence of $\tilde{S}$, the probability that the loss to the CCP exceeds (a given) $S > 0$ and members prefunded and unfunded DF assets is given by

$$P(\tilde{L}_0^j > 0) = q\left[1 + \left((\frac{q}{q_D})^{1/\alpha} - 1\right)\left(\frac{(1+\beta)D}{E_j} + \frac{S}{E_j} - \beta c_1\right)\right]^{-\alpha}. \tag{49}$$

Proposition 4.4 shows that the likelihood that the loss to the CCP exceeds S and members prefunded and unfunded DF assets takes its maximum conditional on the default of member 1 — the member to which the CCP has the largest exposure. Similarly, the probability that the loss to a non-defaulting member exceeds its prefunded DF assets is largest conditional on the default of member 1.

Suppose that S is formulated according to (46). When $\tilde{S} = 0$, we can write $P(\tilde{L}_i^1 > D_i) \le \pi \le q_D$. We denote the upper bound for $P(\tilde{L}_i^1 > D_i)$ that corresponds to $\tilde{S} = 0$ and S formulated as in (46) by $\tilde{\pi}_0$. For instance, when $\pi = q_D$, i.e., when $S_l = (1 - c_1)D$, setting $\tilde{S} = 0$ gives

$$\tilde{\pi}_0 = q\left[1 + \left((\frac{q}{q_D})^{1/\alpha} - 1\right)\left(2 - c_1\right)\right]^{-\alpha}. \tag{50}$$

[27] In the context of CCP equity capital regulation, when regulatory SITG is viewed as the *minimum* amount of equity capital, policymakers could consider S_l instead of S_u.

[28] Proof of Proposition 4.4 is omitted as the method of proof follows that of Propositions 4.2 and 4.3.

This quantity could be interpreted as follows: if all members accept the risk of their DF assets being depleted with probability $\tilde{\pi}_0$, then the second layer SITG will not be required.[29]

Now, working backwards and given our formulation of S with the target loss probability $\pi \leq q_D$, we can specify $\tilde{S}$ that corresponds to a given target loss probability $\tilde{\pi}$, where $\tilde{\pi} \leq \tilde{\pi}_0$. For instance, suppose that $\tilde{\pi} = P(\tilde{L}_i^1 > D_i)$. Solving for $\tilde{S}$ gives

$$P(\tilde{L}_i^1 > D_i) = \tilde{\pi} \iff \tilde{S} = \left(\frac{(\frac{q}{\tilde{\pi}})^{1/\alpha} - (\frac{q}{\pi})^{1/\alpha}}{(\frac{q}{q_D})^{1/\alpha} - 1} + c_1 - 1 \right) D, \tag{51}$$

where $\tilde{\pi} < \tilde{\pi}_0 \leq q_D$.[30] Note that setting $\pi = q_D$ and so $S_l = (1 - c_1)D$ gives

$$P(\tilde{L}_i^1 > D_i) = \tilde{\pi} \iff \tilde{S} = \left(\frac{(\frac{q}{\tilde{\pi}})^{1/\alpha} - 1}{(\frac{q}{q_D})^{1/\alpha} - 1} + c_1 - 2 \right) D. \tag{52}$$

In our Pareto-based framework, designing S and $\tilde{S}$ with target loss probabilities π and $\tilde{\pi}$ ensure that the basic and overarching incentive compatibility constraint (30) is satisfied,

$$P(\tilde{L}_i^j > D_i) \leq \underbrace{P(\tilde{L}_i^1 > D_i)}_{\tilde{\pi}} < q_D,$$

for any $j \neq i, 1$. In sum, our formulation of S and $\tilde{S}$ in this section achieves the incentive compatibility constraints (20), (23), and (30) developed earlier and will result in

$$P(\tilde{L}_i^j > D_i) < P(L_0^j > 0) \leq P(L_i^j > 0) \leq q_D. \tag{53}$$

The last two inequalities follow from our results in the previous section, and the first inequality follows from the definitions of $\tilde{L}_i^j$ and L_0^j.

Remark It is important to note that under Assumption 4.1, $\tilde{S}_l$ and $\tilde{S}_u$, which were introduced in the previous section, can be viewed as special cases of our general Pareto-based framework. To see this, consider scenario A in the previous section and note that (51) can also be written as

[29]We note that $\tilde{\pi}_0$ is an increasing function of c_1, achieving its hypothetical minimum at $c_1 = 0$.

[30]In the formulation of $\tilde{S}$, the first term on the right side is positive because the target probability $\tilde{\pi}$ is set to be less than $\tilde{\pi}_0$.

$$P(\tilde{L}_i^1 > D_i) = \tilde{\pi} \iff \tilde{S} = \left(\frac{(\frac{q}{\tilde{\pi}})^{1/\alpha} - (\frac{q}{\pi})^{1/\alpha}}{(\frac{q}{q_D})^{1/\alpha} - 1} - 1 \right) D - S,$$

for any $S > 0$. When the target loss probability $\tilde{\pi}$ is set as $\tilde{\pi} = P(\tilde{L}_0^1 > 0)$, then Proposition 4.4 and the above formulation results in $\tilde{S}_l = \beta D(1 - c_1)$. Now, consider scenario B where the specific incentive compatibility objective is satisfying (38). It is clear that $\tilde{S}_u = \beta D(1 - c_N)$ satisfies (38). It is also clear that under $\tilde{S}_u$, the basic ICC (30) holds with $P(\tilde{L}_i^1 > D_i) = \tilde{\pi}$.

4.5 Cover-n Case

Appendix A.5 extends our results to the case of the prefunded default fund sized to cover n member defaults; $n \geq 2$. In what follows, we present our SITG formulations under cover-n DF. The total DF is represented by $D_{s,n} = E_1 + E_2 + ... + E_n$, and $L_{i,n}^1$ represents the loss of non-defaulting member i conditional on the default of the largest member.[31] Also, S_n ($\tilde{S}_n$) represents the first (second) layer of SITG under cover-n DF.

Similar to the earlier analysis, in our Pareto-based framework, we formulate S_n that correspond to the target loss probability $\pi_n = P(L_{i,n}^1 > 0)$.

Under our incentive compatibility framework, the first layer of SITG can be formulated according to (91) in the Appendix,

$$S_n = \left[\left(\frac{(\frac{q}{\pi_n})^{1/\alpha} - 1}{(\frac{q}{q_D})^{1/\alpha} - 1} \right) \left(\frac{c_1}{\sum_{k=1}^n c_k} \right) - c_1 \right] D_{s,n}.$$

and the second layer can be formulated as in (99),

$$\tilde{S}_n = \left[\left(\frac{(\frac{q}{\tilde{\pi}_n})^{1/\alpha} - (\frac{q}{\pi_n})^{1/\alpha}}{(\frac{q}{q_D})^{1/\alpha} - 1} \right) \left(\frac{c_1}{\sum_{k=1}^n c_k} \right) + c_1 - 1 \right] D_{s,n}.$$

We note that $\tilde{\pi}_n$ is the target loss probability associated with the second layer of SITG, $\tilde{\pi}_n = P(\tilde{L}_{i,n}^1 > D_{i,n})$. In the cover-$n$ case, $\tilde{L}_{i,n}^j$ represents the total loss to member i's prefunded and unfunded DF contributions conditional on the default of member j. Member i's predunded DF is denoted by $D_{i,n}$. Our formulations of S_n and $\tilde{S}_n$ under cover-n DF, should be compared with S and $\tilde{S}$ under cover-one DF stated in (46) and (51).

In our framework, the first and second layer of SITG can be expressed as a percentage of total DF. This quantity is publicly available from CCPs' quantitative disclosure data. The concentration ratio can also be approxi-

[31] Consistent with our analysis so far, default losses from the perspective of the CCP and members are conditional on the default of a single member. It is not difficult to carry out the analysis conditional on $n \geq 2$ simultaneous defaults.

mated from publicly available data. Recall that under Lemma A.2, when tail exposures are modeled by Student-t distributions, we can write $M_1/(\sum_{k=1}^n M_k) = E_1/(\sum_{k=1}^n E_k) = c_1/(\sum_{k=1}^n c_k)$. Loss probabilities q and q_D that correspond to IM and DF are often prescribed by the regulators. As discussed earlier, the tail exponent α is often in the $2-4$ range. Our incentive compatibility structure can mitigate CCP risk management incentive distortions to varying quantifiable degrees by properly specifying target loss probabilities π_n and $\tilde{\pi}_n$.

5 Capital Regulation

Using the EVT (Pareto)-based framework, we first introduce a lower bound for CCP capital requirements and give a numerical example to show how it varies as a percentage of DF. Next, our framework is used to test the adequacy of Basel CCP risk capital rules.

5.1 Minimum CCP Capital Requirements

As discussed earlier, the CCP's equity capital contribution to the default waterfall could be viewed as a lower bound on its regulatory capital. Our framework gives the following lower bound for CCP minimum capital requirements,

$$S + \tilde{S} = \left[\frac{(\frac{q}{\tilde{\pi}})^{1/\alpha} - 1}{(\frac{q}{q_D})^{1/\alpha} - 1} - 1 \right] D, \tag{54}$$

where S and $\tilde{S}$ are the first and second layer SITG given in (46) and (51). Also, recall that the first target loss probability, $\pi = P(L_i^1 > 0)$ and the second target loss probability $\tilde{\pi} = P(\tilde{L}_i^1 > D_i)$ satisfy $\tilde{\pi} < \pi \leq q_D < q$. As discussed earlier, a simple way to ensure that $\tilde{S} > 0$ is by setting $\tilde{\pi} < \tilde{\pi}_0$, where $\tilde{\pi}_0$ given in (50), and it is derived under $\tilde{S} = 0$ and $S = (1 - c_1)D$.

It is not difficult to show that as the number of members increases, $S + \tilde{S}$ increases under our framework. To see this, note that $\tilde{\pi}_0$ is an increasing function of c_1. So, when the number of members increases, c_1 decreases, this would reduce $\tilde{\pi}_0$, which in turn lowers the target loss probability $\tilde{\pi}$ in our SITG formulation. And, when $\tilde{\pi}$ decreases, the total level of $S + \tilde{S}$ increases. In words, all esle qual, as the number of members increases, higher levels of SITG are required to mitigate risk management agency problems. This can be contrasted with the work of Kuong and Maurin (2023), where a main implication of their contract-theoretic model is rationalizing the documented empirical observations that larger CCPs allocate smaller amounts of their own capital to the default waterfall.

It is worth emphasizing that our proposed CCP regulatory capital lower bound (54) is grounded on the incentive compatibility framework developed earlier. It also only depends on parameters and variables easily accessible to and monitored and controlled by CCPs and their regulators. The following example can be insightful as it shows how the proposed lower bound varies as a function of $\tilde{\pi}, q_D, q$, and α.

Example 5.1 Table 1 reports the ratio of the lower bound on regulatory capital to DF for different values of α, q, q_D, and $\tilde{\pi}$. First, note that D is governed by the confidence level $1 - q_D$ used to define default fund stress scenarios. Note that D is a decreasing function of q_D. So, all else equal, $(S + \tilde{S})/D$ can be viewed as an increasing function of q_D.

Recall that $\tilde{\pi} < \tilde{\pi}_0$ should hold to have $\tilde{S} > 0$. Table 1 reports values of $\tilde{\pi}$ for which there exists $c_1 \in (0, 1)$ and so $\tilde{\pi}_0 \in (0, q_D)$ where $\tilde{\pi} < \tilde{\pi}_0$ holds. For instance, consider the first row of Table 1, with $\alpha = 2$, $q = .01$, and $q_D = .005$. Setting $c_1 = .01$ gives $\tilde{\pi}_0 = .003$ while setting $c_1 = .9$ gives $\tilde{\pi}_0 = .0047$. So, there exists $\tilde{\pi}_0$ for which $\tilde{\pi} = .0035$ falls below $\tilde{\pi}_0$.

In interpreting Table 1, it is also useful to view α, q, and q_D as given and note that higher target loss probabilities $\tilde{\pi}$ are generally associated with lower $(S + \tilde{S})/D$ ratios. In other words, to obtain lower loss probabilities, the lower bound on the regulatory capital $S + \tilde{S}$ should be formulated as a higher percentage of default fund D. For instance, consider the two rows associated with $\alpha = 5$. To achieve $\tilde{\pi} = .0045$, the lower bound on capital requirements becomes $.16D$ while lowering this loss probability by roughly 22 percent to $\tilde{\pi} = .0035$ requires more than 250 percent increase in CCP capital contributions, $S + \tilde{S} = .57\ D$.

As discussed in Section 1, this numerical example shows that a total level of SITG below 15-20 percent of DF cannot be produced in the realistic part of our model parameter space. This contrasts with the current practice that has been empirically summarized in the two recent *Risk.net* articles by Thiruchelvam and Walker. The ratio of total IM to total DF may also vary widely across CCPs, IM can be 10 times larger than DF (Ghamami and Glasserman, 2017). Using 2015-2017 CCP quantitative disclosure data, Huang (2019) estimates an average SITG of about USD 38 million and an average IM of about USD 14 billion across 9-10 investor-owned CCPs. When DF is 10 percent of IM, SITG becomes 2.7 percent of DF. Under our framework, this level of SITG may not adequately mitigate CCP risk management incentive distortions.

The current regulatory regime is not risk-based, (it is not based on economic or financial economics analysis), our framework could be used to improve CCP capital regulation.

Remark The CCP regulatory capital lower bound in the more general cover-n DF setting is derived in Appendix A.5. Systemically important

$\frac{S+\tilde{S}}{D}$	α	q (bps)	q_D (bps)	$\tilde{\pi}$ (bps)
0.67	2	100	50	35
0.18	2	100	50	45
0.09	2	200	100	95
1.62	2	100	30	10
2.51	2	100	80	50
17.32	2	100	80	10
3.44	3	100	50	10
0.61	3	100	50	35
1.34	3	100	30	10
0.36	4	100	50	40
0.59	4	100	50	35
0.67	4	80	60	50
3.17	4	70	40	10
0.3	4	100	80	75
0.16	5	100	50	45
0.57	5	100	50	35
2.93	6	100	50	10
0.57	6	100	50	35
0.62	6	100	80	70

Table 1: Total SITG, $S+\tilde{S}$, as a fraction of default fund D, computed from (54), for different values of parameters α, q, q_D, and $\tilde{\pi}$.

derivatives CCPs often operate under cover-2 DF, where the lower bound becomes

$$S_2 + \tilde{S}_2 = \left[\left(\frac{(\frac{q}{\tilde{\pi}_2})^{1/\alpha} - 1}{(\frac{q}{q_D})^{1/\alpha} - 1} \right) \left(\frac{c_1}{c_1 + c_2} \right) - 1 \right] D_{s,2}. \tag{55}$$

This follows from (100) in the Appendix. Recall that in the cover-one case, we have $D = E_1$, and total DF is $D_{s,2} = E_1 + E_2$ in the cover-two case.[32] It is important to note that under similar incentive compatibility structures, we have $\tilde{\pi}_2 < \tilde{\pi}$. This can result in $S_2 + \tilde{S}_2 \geq S + \tilde{S}$.

Optimal Capital Regulation

The lower bound on CCP equity capital (54) focuses on incentive compatibility constraints and central clearing risk management agency problems. It abstracts away from costs of CCP capital and CCP failure. Consider the

[32] As described in Appendix A.5, in the more general cover-n DF setting, we have further simplified the notation by assuming that $E_N \leq E_{N-1} \leq ... \leq E_2 \leq E_1$. That is, under the cover-n rule, we write $D = \sum_{i=1}^{n} E_n$.

case of a single representative CCP. Recall our sketch of the CCP objective function under outside ownership (8). Social welfare can be represented by

$$\phi V - E[L] - E[L_e] - c(E_t) - c_s Q(S(\pi), \tilde{S}(\tilde{\pi}), E_s), \tag{56}$$

where under the incentive compatibility framework, the first layer of SITG, $S(\pi)$, can be viewed as a decreasing function of the target loss probability π, and given π, the second layer of SITG, $\tilde{S}(\tilde{\pi})$, is a decreasing function of the second target loss probability $\tilde{\pi}$. Given members' initial margin and default fund assets, the social planner's objective would then be to find the optimum SITG and E_s that maximize (56). In addition to IM and DF, if the Pareto tail exponent, α, is also given, the social planner's problem could be equivalently viewed as finding the optimum π, $\tilde{\pi}$, and E_s that maximize (56). Optimal SITG levels could then correspond to target loss probabilities under which some of the ICCs may be satisfied and some may not hold from members' perspective. This paper does not study optimal SITG levels. Instead, our focus has been on developing an incentive compatibility framework that incorporates SITG into policymakers' *revised objective function*. That is, given the CCPs' default waterfall, we replace the social planner's inaccurate objective function (10) with a more accurate one (56) and link S and $\tilde{S}$ to a set of incentive compatibility constraints under which some of the CCP risk management agency problems can be mitigated.

5.2 CCP Risk Capital

CCP risk capital refers to capital requirements for bank exposures to CCPs (BCBS, 2023). Policymakers sometimes borrow ideas from bank capital regulation when regulating CCPs. For instance, in the context of credit and counterparty risk capital, a CCP is viewed as a financial firm holding portfolios of financial assets with N counterparties. The CCP's minimum risk-based capital requirement is then a percentage of its *risk weighted assets*, where the risk weights represent the credit quality (default probability) of members (counterparties) and assets represent the CCP's exposure to its members net IM and DF over a given time period, (Ghamami, 2015). Adopting the classical bank capital regulation framework for CCP capital regulation can be seen in the Basel Committee's formulation of CCP risk capital. In our setting, the Basel Committee's *hypothetical* CCP capital requirement is formulated as

$$K_{ccp} = k_r \times \sum_{i=1}^{N} E[U_i - M_i - D_i]^+ \omega_i, \tag{57}$$

where the capital ratio k_r is set to 8%, and the minimum requirement for risk weights ω_i is equal to 20%. K_{ccp} with this fixed risk weight is then used to formulate the CCP risk capital rule. More specifically, the amount of capital that a bank needs to hold against its exposure to a CCP is an increasing function of K_{ccp}.

Since the sum of first and second layers of SITG can be viewed as a lower bound for CCP capital requirements, there are different ways where $S + \tilde{S}$ can be used to test the adequacy of CCP risk capital. The following approach will be insightful. Note that if the inequality

$$K_{ccp} \geq S + \tilde{S}, \tag{58}$$

does not hold, regulators may need to revisit the definition or formulation of K_{ccp}, adjust k_r or ω, or they can replace K_{ccp} with $S+\tilde{S}$. Since ω represents the average credit quality of clearing members, we have $0 \leq \omega \leq 1$. This suggests that $0 \leq \omega k_r \leq 1$. Consequently, if

$$S + \tilde{S} \geq \sum_{i=1}^{N} E[U_i - M_i - D_i]^+, \tag{59}$$

holds under our framework, policymakers can revisit CCP risk capital. That is, our framework implies that under (59), the CCP risk capital rule can underestimate bank exposures to CCPs. Now, consider the following ratio,

$$R = \frac{S + \tilde{S}}{\sum_{i=1}^{N} E[U_i - M_i - D_i]^+}. \tag{60}$$

Proposition 5.2 derives a useful expression for R.

Proposition 5.2 *Under Assumption 4.1, the lower bound* (60) *for the ratio of SITG to Basel CCP capital,* K_{CCP}*, is given by*

$$R = q^{-1}c_1(\alpha - 1)\left[(\frac{q}{\tilde{\pi}})^{1/\alpha} - (\frac{q}{q_D})^{1/\alpha}\right]\left[1 + c_1\left((\frac{q}{q_D})^{1/\alpha} - 1\right)\right]^{\alpha-1}. \tag{61}$$

The following numerical example shows that condition (59) holds in most parts of the model parameter space.

Example 5.3 Table 2 reports R in (61) for different values of the tail index α, concentration ratio c_1, loss probabilities associated with IM and DF confidence levels, q and q_D, and member target loss probabilities associated with unfunded DF, $\tilde{\pi}$. As discussed earlier, to have $\tilde{S} > 0$, the target loss probability $\tilde{\pi}$ should satisfy $\tilde{\pi} < \tilde{\pi}_0$, where $\tilde{\pi}_0$ is defined in (50).[33] Note that

[33]Recall that $\tilde{\pi}_0$ is the loss probability associated with unfunded DF when we set $S = (1 - c_1)D$ and $\tilde{S} = 0$.

R	α	c_1	q (bps)	q_D(bps)	$\tilde{\pi}_0$ (bps)	$\tilde{\pi}$(bps)
4.19	2	.05	100	50	31	20
2.10	2	.05	100	50	31	30
8.56	2	.10	100	50	31	20
1.08	2	.10	80	70	63	60
4.56	2	.15	100	70	54	45
37.86	2	.20	100	50	33	10
7.66	2	.20	100	50	33	32
58.96	2	.30	100	50	34	10
18.83	3	.10	100	50	30	10
5.28	3	.10	100	50	30	29
39.6	3	.20	100	50	32	10
3.85	3	.20	70	60	50	53
.66	*3*	*.01*	*100*	*50*	*29*	*25*
16.6	4	.20	100	40	22	20
68.19	4	.40	100	20	9.69	9
4.81	4	.15	90	70	57	50
1.76	5	.01	100	50	27	10
.69	*5*	*.01*	*100*	*50*	*27*	*25*
11.19	5	.15	100	50	30	25
8.66	5	.15	100	50	30	29
.69	*6*	*.01*	*100*	*50*	*27*	*25*
4.31	6	.10	100	80	66	50
7.30	6	.10	100	50	29	25

Table 2: Lower bound R for the ratio of SITG to K_{ccp} for different values of α, c_1, q, q_D, $\tilde{\pi}_0$, and $\tilde{\pi}$. We note that $R > 1$ in most cases unless the concentration ratio is unrealistically small and $\tilde{\pi}$ is close to $\tilde{\pi}_0$, which gives $\tilde{S}$ close to zero. Rows in italics report the part of the parameter space producing $R < 1$.

R is an increasing function of c_1. It is clear that $R > 1$ in most parts of the parameter space unless the concentration ratio is very small and $\tilde{\pi}$ is close to $\tilde{\pi}_0$, giving values of $\tilde{S}$ close to zero.

In short, Proposition 5.2 along with the above numerical example indicate that capital rules on banks due to their exposures to CCPs may not sufficiently absorb central clearing risks.

6 Monolayer Default Waterfall

Large derivatives CCPs do not often mutualize the pool of initial margin to cover defaulting member losses. It is the default fund, the layer of collateral collected in addition to IM, that can be mutualized to cover losses.

The default waterfall in some CCPs, particularly at the largest securities clearinghouses in the U.S. could be different. Securities CCPs in the U.S. often collect IM from their members and mutualize the pool of IM to cover losses. Total IM under the monolayer default waterfall plays the role of DF in CCPs with the more typical multilayered waterfall.

In this section, we analyze the monolayer default waterfall by modeling and estimating losses from the perspective of a CCP and its members. We show that compared to CCPs with the multilayered waterfall structure, clearinghouses with monolayer default waterfall may need to have significantly higher levels of SITG to mitigate moral hazard problems linked to risk management incentives.

Monolayer CCPs have become increasingly important as a broader central clearing mandate is a critical element of the government securities market reform programs initiated by G30's Working Group on Treasury Market Liquidity after the emergence of the Covid-19 pandemic.[34]

6.1 CCP Perspective

In the absence of SITG, the exposure of a monolayer CCP conditional on the default of member j can be written as

$$\check{L}_0^j = (U_j - M)^+,$$

where $M = \sum_{i=1}^N M_i$, and $j = 1, ..., N$. To compare monolayer and multilayered default waterfalls, we make the natural assumption that U_j, the random variables representing the CCP's exposure to member defaults, are drawn from the same distribution under both waterfall structures. Recall that the CCP's exposure to the default of a member under the more typical multilayered waterfall structure is written as $L_0^j = (U_j - M_j - D)^+$. As will be explained shortly, in practice, it is often the case that

$$P(\check{L}_0^j > x) \leq P(L_0^j > x), \tag{62}$$

for any $x \geq 0$. In words, the monolayer CCP is less exposed to the default of its members than the multilayered CCP. It is well-known that in CCPs operating under the multilayered default waterfall, total IM can be notably larger than the total DF. In fact, M can be more than 10 times larger that D, particularly in large CCPs, (Ghamami and Glasserman, 2017).[35] Note

[34]See the original G30 report G30 (2021) and the subsequent Status Update report G30 (2022).

[35]According to the Global Association of Central Counterparties, the total required IM for selected 24 CCPs was 956.6 billion USD, and the total required DF was 49.1 billion USD in 2022Q4. Aggregated over 24 CCPs, the ratio of IM to DF was around 19.5 at the end of 2022. See the 2022 Annual Markets Review in Central Counterparty Clearing.

that (62) holds when

$$M - M_j \geq D, \tag{63}$$

for any $j = 1, ..., N$. This condition is satisfied unless a CCP has an unrealistically high concentration ratio c_1. It should be intuitively clear that since collateral in the form IM often dominates DF, sharing IM to cover losses could be beneficial from the CCP's perspective.[36]

6.2 Member Perspective

It is also straightforward to see that compared to multilayered CCPs, in monolayer CCPs, surviving members are more exposed to the defaulting member losses. Note that SITG under the monolayer waterfall structure, denoted by $\check{S}$, comes into play right after the defaulters' IM. Conditional on the default of member j, potential losses to the IM contribution of surviving member i can be written as

$$(U_j - M_j - \check{S})^+ \frac{M_i}{M - M_j},$$

where $i \neq j$. In the scenario where surviving member losses cannot exceed M_i, we have

$$\check{L}_i^j = M_i \min\left(\frac{(U_j - M_j - \check{S})^+}{M - M_j}, 1\right). \tag{64}$$

Comparing the two waterfall structures, we now show that members could incur larger losses when the IM pool is mutualized in the absence of an additional and separate prefunded default fund.

Proposition 6.1 *If the SITG is sized similarly under both types of waterfall structures, we have*

$$P(\check{L}_i^j > 0) > P(L_i^j > 0), \tag{65}$$

where $i \neq j$, *and* $\check{L}_i^j$ *and* L_i^j *are defined in* (64) *and* (13), *respectively. Let* σ_i^2 *denote the variance of* U_i. *Furthermore if* $U_i/\sigma_i \sim T(0, \nu)$ *has a mean-zero Student- t distribution with* $\nu > 1$ *degrees of freedom then for any loss level* $x > 0$

$$P(\check{L}_i^j > x) > P(L_i^j > x). \tag{66}$$

[36]Our analysis does not take into account recovery schemes such as variation margin haircuts (Cont, 2015).

Remark Student-t distributions belong to the class of elliptical distributions that encompasses a large battery of models used in finance and economics, particularly in risk management (Andersen and Dickinson (2018); Cont and Kan (2011); Ivanov (2017); Ghamami and Glasserman (2019)). Student-t distributions include the normal distribution as a limiting case (for $\nu \to \infty$). Since U_i, the CCP's exposure to member i, captures in part portfolio value changes over the MPOR whose length is often five days or less, it is not unrealistic to assume that U_i has mean zero.

6.3 CCP Capital Contribution

If the CCP does not allocate its own capital to the default waterfall,[37] we have

$$P(\check{L}_i^j > 0) = P(U_j > M_j) = q,$$

where $1 - q$ is the confidence level associated with the VaR-based initial margin. That is, conditional on any member's default, the probability that a surviving member's IM would incur losses is q. Comparing this member loss probability q with CCP's loss probability conditional on a member's default

$$P(\check{L}_0^j > 0) = P(U_j > M_j + \sum_{i \neq j} M_i),$$

it is clear that members could incur disproportionately larger losses compared to the CCP. To perfectly align largest counterparty default loss probabilities between the CCP and its members, the CCP should contribute $\sum_{i=2}^N M_i$ to the loss waterfall. That is, setting $\check{S} = M - M_1$, gives

$$P(\check{L}_i^1 > 0) = P(\check{L}_0^1 > 0), \tag{67}$$

for any $i \neq 1$. Note that condition (67) is analogous to ICC (23) that is satisfied under (21) in multilayered CCP. We will return to this formualtion of SITG toward the end of this section.

We now use our Pareto-based framework to draw a useful comparison between monolayer and multilayered default waterfalls. We continue to assume that exposures U_i are drawn from similar distributions, and that E_i are quantified based on a given confidence level $1 - q_D$ under both waterfall structures. Using Assumption 4.1 and employing the method of proof in Proposition 4.3, we have

[37] Securities CCPs often have only one layer of SITG, which is denoted by $\check{S}$ in our analysis. It is not difficult to extend our analysis to formulate the second layer of SITG for monolayer CCPs.

$$P(\check{L}_i^j > 0) = q\left[1 + \left((\frac{q}{q_D})^{1/\alpha} - 1\right)\frac{\check{S}}{E_j}\right]^{-\alpha},$$

which implies that $P(\check{L}_i^j > 0) \leq P(\check{L}_i^1 > 0)$ for any $j \neq i, 1$. We now specify SITG corresponding to a given target loss probability $\breve{\pi}$. For instance, suppose that $\breve{\pi} = P(\check{L}_i^1 > 0)$, where $\breve{\pi} \leq q_D$, associated with the largest member default. Solving for $\check{S}$ yields,

$$P(\check{L}_i^1 > 0) = \breve{\pi} \iff \check{S} = \left(\frac{(\frac{q}{\breve{\pi}})^{1/\alpha} - 1}{(\frac{q}{q_D})^{1/\alpha} - 1}\right)E_1. \tag{68}$$

Choosing $\breve{\pi} = q_D$ simplifies the above equation to

$$\check{S}_{q_D} = E_1. \tag{69}$$

That is, $\check{S}_{q_D} = D$ unde the cover-one DF rule. Consequently, if SITG is formulated according to (69), then in the event of the default of the largest member, the probability that a surviving member would incur any losses will be $q_D < q$. This formulation of SITG is insightful as it provides a way to directly compare monolayer and multilayered CCPs in terms of the required capital contribution to the waterfall that would guarantee similar default exposures from the perspective of members. That is, the two waterfall structures result in equal member loss probabilities (conditional on the largest member default),

$$P(\check{L}_i^1 > 0) = P(L_i^1 > 0) = q_D < q,$$

when the monolayer CCP operates under $\check{S}_{q_D} = D$ while the multilayered CCP contributes $S = (1 - c_1)D$. Given that $0 < c_1 < 1$, this results shows that policymakers may need to require higher levels of SITG at monolayer CCPs so members would be exposed to similar levels of default risk under both types of waterfall structures. All else equal, the higher the concentration ratio, the larger the difference between CCP equity capital contributions under monolayer and multilayered default waterfalls.

The incentive compatibility framework can also be used more directly to compare monolayer and multilayered CCPs. Recall ICC (23) under the multilayered default waterfall in scenario A. The analogous ICC for the monolayer CCP could be written as

$$P(\check{L}_i^1 > 0) = P(\check{L}_0^1 > 0) = \breve{\pi}. \tag{70}$$

As discussed earlier, the above ICC is satisfied under $\check{S} = M - M_1$, which can be written as

$$\check{S}_l = (1 - c_1)M \tag{71}$$

by Lemma A.2 when we assume that $U_i/\sigma_i \sim T(0, \nu)$ has a mean-zero Student- t distribution with $\nu > 1$ degrees of freedom. We note that under (71), the following basic and overarching ICC in the monolayer case is also satisfied,

$$P(\check{L}_i^j > 0) \leq P(\check{L}_0^1 > 0), \tag{72}$$

for any $j \neq i, 1$. It is important to note that we just provided economic arguments initially used in the design of SITG that led to $S_l = (1 - c_1)D$ under the multilayered waterfall. In short, when normalized exposures U_i have heavy (Pareto) tails, incentive compatibility constraints require

$$\check{S}_l = \frac{M}{D} S_l. \tag{73}$$

In words, the capital contribution of the monolayer CCP to the default waterfall may need to be several multiples of that of a similar CCP with a multilayered waterfall.

To complete our comparative analysis of incentive structures associated with monolayer and multilayered CCPs, recall scenario B under the more typical default waterfall where $S_u = (1 - c_N)D$ would guarantee that ICC (26) holds. We note that setting

$$\check{S}_u = M - M_N = (1 - c_N)M \tag{74}$$

leads to a similar incentive structure for the monolayer CCP: under (74), the analogous ICC

$$P(\check{L}_i^j > 0) \leq P(\check{L}_0^j > 0), \tag{75}$$

holds for any $j \neq i, 1$. Consequently, when enforcing this second incentive structure for monolayer and multilayered CCPs, we arrive at the same result (73). That is, the SITG of the monolayer CCP would need to be M/D times larger than the SITG under the multilayered default waterfall.

In summary, our results illustrate that compared to multilayered clearinghouses, CCPs operating under the monolayer structure should allocate more capital to the default waterfall.

7 Concluding Remarks

Recent distress in the banking sector highlights the fact that post-GFC recovery and resolution frameworks may not work well in practice and may need to be improved. The Silicon Valley Bank (SVB) collapsed in March 2023. While SVB was subject to bank-level resolution planning, and most of its assets were held in the bank, the resolution plans could not be implemented successfully (Clancy, 2023). Effective CCP capital and SITG regulation is at least as important as improving CCP recovery and resolution frameworks.

We have proposed a robust and objective framework that can be used for designing CCP SITG requirements. Our framework is grounded in incentive compatibility constraints that capture central clearing risk management agency problems. The proposed SITG formulae are simple and readily implementable using data available to CCPs and regulators. Comparing our SITG formulations with CCP public data and the empirical evidence from recent CCP quantitative disclosures (Ghamami and Glasserman (2017); Huang (2019); Thiruchelvam (2022); Walker (2023)), we conclude that investor-owned and member-owned CCPs may need to allocate more capital to default waterfalls.

Central clearing will play a key role in the reform of U.S. Treasury market. Resilience of clearinghouses that will be at the center of the UST market is of critical importance.[38] To diversify the supply of Treasury market liquidity under stress, the first recommendation of G30 (2021) was that the Federal Reserve should create a Standing Repo Facility (SRF) that provides very broad access to repo financing for U.S. Treasury securities on adequate terms. The SRF that the Fed created in 2021 did not provide the very broad access recommended in (G30, 2021). This could have been in part due to concerns about creating moral hazard problems that would increase systemic risks as broader SRF access may incentivize firms to increase their leverage (G30, 2022). The G30 have suggested that this moral hazard could be mitigated by centrally clearing repos provided by the SRF. Our investigation indicates that this agency problem may be counteracted when CCP risk management agency problems are mitigated effectively.

[38]G30 (2021) emphasize that concerns about the concentration of risk at CCPs and their transparency and governance should be addressed properly.

A Appendix

A.1 Proof of Proposition 4.2

Note that

$$P(L_i^1 > x) = P\left(U_1 > M_1 + D_1 + S + \frac{x(D - D_1)}{D_i}\right). \tag{76}$$

Since $D_i = c_i D$, we can write

$$P(L_i^1 > x) = P\left(U_1 > M_1 + D_1 + S + \frac{x(1 - c_1)}{c_i}\right).$$

Set $A \equiv D_1 + S + \frac{x(1-c_1)}{c_i}$. The probability on the right side above can be written as

$$P(U_1 - M_1 > A) = qP(U_1 - M_1 > A|U_1 > M_1), \tag{77}$$

where $q = P(U_1 > M_1)$. Recall that $(U_1 - M_1)|U_1 > M_1 \sim \mathrm{Pa}(\alpha, \kappa_1)$, and so,

$$P(U_1 - M_1 > A|U_1 > M_1) = \left(\frac{\kappa_1 + A}{\kappa_1}\right)^{-\alpha}. \tag{78}$$

We now remove the dependence of the term on the right side above on κ_1. To do so, note that

$$q_D = P(U_1 > M_1 + D) = qP\left(U_1 > M_1 + D|U_1 > M_1\right).$$

Again, using our modeling assumption $(U_1 - M_1)|U_1 > M_1 \sim \mathrm{Pa}(\alpha, \kappa_1)$, we can write

$$P\left(U_1 > M_1 + D|U_1 > M_1\right) = \left(\frac{\kappa_1 + D}{\kappa_1}\right)^{-\alpha},$$

and so

$$\frac{D}{\kappa_1} = (\frac{q}{q_D})^{1/\alpha} - 1. \tag{79}$$

We note that (77)-(79) give (45). This completes the proof.

A.2 Proof of Proposition 4.3

The proof is similar to the proof of Proposition 4.2, we only need to modify the last part of the proof of Proposition 4.2 as follows. Note that

$$P(U_j > M_j + E_j) = q_D = qP(U_j > M_j + E_j | U_j > M_j) = q\left(\frac{\kappa_j + E_j}{\kappa_j}\right)^{-\alpha}.$$

This gives

$$\left(\frac{q}{q_D}\right)^{1/\alpha} - 1 = \frac{E_j}{\kappa_j}, \qquad \text{which leads to} \quad \kappa_j = \frac{E_j}{(\frac{q}{q_D})^{1/\alpha} - 1}.$$

This lead to (47) as its right side does not depend on κ_j.

A.3 Proof of Proposition 5.2

We use the following Lemma to prove Proposition 5.2.

Lemma A.1 *Under Assumption 4.1, we have*

$$E[(U_1 - M_1 - W)^+] = \frac{q\kappa_1}{\alpha - 1}\left(\frac{\kappa_1 + W}{\kappa_1}\right)^{-\alpha+1}. \tag{80}$$

where $W > 0$ *is a constant.*

Proof of Lemma A.1

Note that

$$E[(U_1 - M_1 - W)^+] = E[(U_1 - M_1)1_A] - WP(A) \tag{81}$$

where 1_A is the indicator of the event $A = \{U_1 - M_1 > W\}$. First, consider the expectation on the right side above. The conditional probability density function of $(U_1 - M_1)|U_1 > M_1$ is

$$f_1(u) = \frac{\alpha}{\kappa_1}\left(\frac{\kappa_1 + u}{\kappa_1}\right)^{-\alpha-1},$$

where $u \geq 0$. Note that

$$E[(U_1 - M_1)1_A] = qE\left[(U_1 - M_1)1_A \middle| U_1 > M_1\right].$$

We use integration by parts to calculate the conditional expectation above to derive

$$E[(U_1 - M_1)1_A] = q\int_W^\infty uf_1(u)du = q\left(\frac{\alpha W + \kappa_1}{\alpha - 1}\right)\left(\frac{W + \kappa_1}{\kappa_1}\right)^{-\alpha}. \quad (82)$$

Next, consider the last term on the right side of (81) and note that

$$P(U_1 - M_1 > W) = q\left(\frac{\kappa_1 + W}{\kappa_1}\right)^{-\alpha}. \quad (83)$$

It is straightforward to see that (82) and (83) give (80). This completes the proof.

A.3.1 Proof of Proposition 5.2

Using Assumption 4.1, we can write

$$P(U_i - M_i > E_i) = q_D = q\left(\frac{\kappa_i + E_i}{\kappa_i}\right)^{-\alpha},$$

and so we have

$$\kappa_i = \frac{E_i}{(q/q_D)^{1/\alpha} - 1}.$$

Lemma A.1 and the above expression for κ_i give

$$\sum_{i=1}^N E[U_i - M_i - D_i]^+ = \frac{Eq\left[1 + c_1(\frac{q}{q_D})^{1/\alpha} - 1\Big)\right]^{-\alpha+1}}{(\alpha - 1)\left[(\frac{q}{q_D})^{1/\alpha} - 1\right]}, \quad (84)$$

where $E = \sum_{i=1}^N E_i$. Given our formulations of S and $\tilde{S}$ in (4.2) and (52), we have derived (54). Dividing the right side of (54) by the right side of (84) gives (61). This completes the proof.

A.4 Proof of Proposition 6.1

Note that

$$P(\check{L}_i^j > x) = P\left(U_j - M_j > \check{S} + x\frac{M - M_j}{M_i}\right), \quad (85)$$

for any $x \geq 0$ and $i \neq j$. Now, consider the more typical multilayered waterfall (in the presence of a separate prefunded default fund, where the IM pool is not mutualized). Conditional on the default of member j, the probability distribution of losses to member i's default fund contribution can be written as

$$P(L_i^j > x) = P\left(U_j - M_j > D_j + S + x\frac{D - D_j}{D_i}\right).$$

Clearly, when U_j are drawn from the same distribution under both waterfall structures, setting $x = 0$ and $S = \check{S}$ gives (65). This completes the first part of the proof. We use the following Lemma for the second part.

Lemma A.2 *If $U_i/\sigma_i \sim T(0, \nu)$ has a mean-zero Student- t distribution with $\nu > 1$ degrees of freedom then*

$$\frac{E_i}{\sum_{j=1}^{N} E_j} = \frac{M_i}{\sum_{j=1}^{N} M_j}, \tag{86}$$

where either VaR or ES is used in calculating M_i and E_i.

It is not difficult to see that Lemma A.2 gives $M_i/(M - M_j) = D_i/(D - D_j)$. Consequently, (66) holds for any $x > 0$ and $i \neq j$ as long as $D_i > 0$. This completes the proof.

Proof of Lemma A.2

The proof uses standard results in the theory of quantitative risk management, (McNeil et al. (2015)).

First, suppose that M_i and E_i are calculated based on VaR. That is, $M_i = \text{VaR}_q(U_i)$ and $E_i = \text{VaR}_{q_D}((U_i - M_i)^+)$ with $q_D < q \leq .01$. Given that $U_i/\sigma_i \sim T(0, \nu)$, it is straightforward to show that $M_i = \sigma_i t_q$, where t_q denotes the inverse of Student t cumulative distribution function with mean zero and degrees of freedom ν evaluated at $1 - q$. This results in

$$\frac{M_i}{\sum_{j=1}^{N} M_j} = \frac{\sigma_i}{\sum_{j=1}^{N} \sigma_j}. \tag{87}$$

Since $M_i = \sigma_i t_q$ and $U_i/\sigma_i \sim T(0, \nu)$, it is straightforward to show that $E_i = \sigma_i(t_{q_D} - t_q)$. Consequently,

$$\frac{E_i}{\sum_{j=1}^{N} E_j} = \frac{\sigma_i}{\sum_{j=1}^{N} \sigma_j}. \tag{88}$$

Second, suppose that M_i and E_i are calculated based on ES. When $U_i/\sigma_i \sim T(0, \nu)$, it is well-known and can be easily shown that,

$$M_i = ES_q(U_i) = \sigma_i \frac{g(t_q)}{q}\left(\frac{\nu + t_q^2}{\nu - 1}\right),$$

where g denotes the Student t probability density function with degrees of freedom ν and mean zero. To calculate $E_i = ES_{q_D}((U_i - M_i)^+)$, we use the following well-known result (McNeil et al., 2015, Ch.2),

$$ES_{q_D}((U_i - M_i)^+) = ES_{q_D}(U_i) - M_i,$$

to derive

$$ES_{q_D}((U_i - M_i)^+)) = \sigma_i \left(\frac{g(t_{qD})}{q_D} \left(\frac{\nu + t_{q_D}^2}{\nu - 1} \right) - \frac{g(t_q)}{q} \left(\frac{\nu + t_q^2}{\nu - 1} \right) \right),$$

So, (87) and (88) hold under ES and Student t distribution. This completes the proof.

A.5 Cover-n Case

We now extend our analysis to the scenario where the prefunded default fund is sized under the cover-n rule; $2 \leq n \leq N$. To simplify the notation, suppose that

$$E_N \leq E_{N-1} \leq ... \leq E_2 \leq E_1.$$

In what follows, when necessary, we append a subscript or superscript n to loss variables and other model components to differentiate the cover-n case from the cover-one analysis presented in the main body of the paper. Under the cover-n DF, we can write

$$D_{s,n} = \sum_{i=1}^{n} E_n.$$

Suppose that DF is allocated to members proportional to E_i, and member i's DF contribution is denoted by $D_{i,n}$.

As before, default losses from both member and the CCP's perspective are conditional on the default of a single member.[39] We note that under the cover-n DF, the basic results of Section 3 remain unchanged. Specifically, (17)-(20) and (27)-(30) continue to hold under the cover-n rule. However, while $P(L_0^1 > 0) = q_D$ under the cover-one DF, we have

$$P(L_{0,n}^1 > 0) = P(U_1 - M_1 > D_{s,n}) < q_D,$$

under the cover-n rule. Also, in the absence of any capital contributions by the CCP, we will have

$$P(\tilde{L}_{i,n}^1 > D_{i,n}) = P(U_1 - M_1 > D_{s,n}) < q_D,$$

[39] It is straightforward to carry out the analysis conditional on $n \geq 2$ simultaneous defaults.

under the cover-n rule.[40]

A.5.1 Pareto-based SITG: First Layer

Under Assumption 4.1, the distribution of $L^1_{i,n}$ can be expressed in terms of E_1. For the cover-n DF, we can derive

$$P(L^1_{i,n} > x) = q\left[1 + \left((\frac{q}{q_D})^{1/\alpha} - 1\right)\left(\sum_{k=1}^{n} c_k + \frac{S_n}{E_1} + \frac{(1-c_1)}{c_i}\frac{x}{E_1}\right)\right]^{-\alpha}. \quad (89)$$

The proof is omitted as it is similar to the proof of Proposition 4.2.

Given (89), the target loss probability of $\pi_n = P(L^1_{i,n} > 0)$, where $\pi_n \leq q_D$, results in the following SITG formulation

$$S_n = \left(\frac{(\frac{q}{\pi_n})^{1/\alpha} - 1}{(\frac{q}{q_D})^{1/\alpha} - 1} - \sum_{k=1}^{n} c_k\right) E_1. \quad (90)$$

Simple algebra gives the following second expression,

$$S_n = \left[\left(\frac{(\frac{q}{\pi_n})^{1/\alpha} - 1}{(\frac{q}{q_D})^{1/\alpha} - 1}\right)\left(\frac{c_1}{\sum_{k=1}^{n} c_k}\right) - c_1\right] D_{s,n}. \quad (91)$$

This formulation is useful as S_n is more explicitly written as a percentage of total DF. Recall that we have formulated SITG as a percentage of total DF in the main body of the paper in the cover-one case. We also note that setting $\pi_n = q_D$ gives

$$S_{q_D,n} = \left(1 - \sum_{k=1}^{n} c_k\right) E_1. \quad (92)$$

Conditional on the default of member $j \neq 1$ and under the cover-n rule, we can derive

$$P(L^j_{i,n} > x) = q\left[1 + \left((\frac{q}{q_D})^{1/\alpha} - 1\right)\left(\sum_{k=1}^{n} c_k + \frac{S_n}{E_j} + \frac{x}{E_j}\frac{(1-c_j)}{c_i}\right)\right]^{-\alpha}. \quad (93)$$

The proof is similar to the proof of Proposition 4.3 and so is omitted.

Comparing (89) and (93), we can write $P(L^j_{i,n} > x) \leq P(L^1_{i,n} > x)$. Consequently, formulating S_n according to (90) with the target loss probability

[40] It is also straightforward to extend our analysis to derive *scenario-B SITG formulations* in this more general setting. That is, our framework could be used to construct ranges for the first and second layer SITG that satisfy a battery of incentive compatibility constraints.

$\pi_n \leq q_D$ gives

$$P(L^j_{0,n} > 0) \leq P(L^j_{i,n} > 0) \leq q_D \tag{94}$$

where $L^j_{0,n} = (U_j - M_j - D_{s,n})^+$. Note that

$$P(L^j_{i,n} > 0) \leq q_D,$$

is the basic and overarching ICC (20) we introduced in the cover-1 DF setting.

A.5.2 Pareto-based SITG: Second Layer

Conditional on the default of member 1, the probability that the loss of member i exceeds its default fund contributions becomes[41]

$$P(\tilde{L}^1_{i,n} > D_{i,n}) = q\left[1 + \left((\frac{q}{q_D})^{1/\alpha} - 1\right)\left(\frac{\sum_{k=1}^n c_k}{c_1} + \frac{S_n}{E_1} + \frac{\tilde{S}_n}{E_1}\right)\right]^{-\alpha}. \tag{95}$$

This is to be compared with the first part of Proposition 4.4 in the cover-one case. Note that with S_n being sized according to (90) and setting $\tilde{S}_n = 0$, we will have $P(\tilde{L}^1_{i,n} > D_{i,n}) \leq \pi_n \leq q_D$. We denote this upper bound corresponding to $\tilde{S}_n = 0$ by $\tilde{\pi}_{0,n}$.

Given S_n as in (90), fixing the second target loss probability $\tilde{\pi}_n$, where $\tilde{\pi}_n \leq \tilde{\pi}_{0,n} \leq \pi_n \leq q_D$, and working backwards, we have

$$\tilde{S}_n = \left[\frac{(\frac{q}{\tilde{\pi}_n})^{1/\alpha} - (\frac{q}{\pi_n})^{1/\alpha}}{(\frac{q}{q_D})^{1/\alpha} - 1} + (1 - \frac{1}{c_1})\sum_{k=1}^n c_k\right]E_1. \tag{96}$$

Note that for the special case where $\pi_n = q_D$, $\tilde{S}_n$ becomes

$$\tilde{S}_n = \left[\frac{(\frac{q}{\tilde{\pi}_n})^{1/\alpha} - 1}{(\frac{q}{q_D})^{1/\alpha} - 1} + (1 - \frac{1}{c_1})\sum_{k=1}^n c_k - 1\right]E_1. \tag{97}$$

Similar to cover-one case, for a CCP that operates under the cover-n rule, our SITG formulations lead to

$$P(\tilde{L}^j_{i,n} > D_{i,n}) < P(L^j_{0,n} > 0) \leq P(L^j_{i,n} > 0) \leq q_D. \tag{98}$$

That is, our Pareto-based formulations of S_n and $\tilde{S}_n$ will lower members' default loss probabilities below q_D.

[41] The proof is similar to the proof of Proposition and so is omitted.

Remark We can write (96) as

$$\tilde{S}_n = \left[\left(\frac{(\frac{q}{\tilde{\pi}_n})^{1/\alpha} - (\frac{q}{\pi_n})^{1/\alpha}}{(\frac{q}{q_D})^{1/\alpha} - 1} \right) \left(\frac{c_1}{\sum_{k=1}^{n} c_k} \right) + c_1 - 1 \right] D_{s,n}. \tag{99}$$

This formulation is useful as it explicitly expresses $\tilde{S}_n$ as a fraction of total prefunded default fund in the cover-n case. In short, it is insightful to compare $\tilde{S}_n$ formulated in (99) with $\tilde{S}$ formulated in (51) in the cover-one case. We also note that

$$S_n + \tilde{S}_n = \left[\left(\frac{(\frac{q}{\tilde{\pi}_n})^{1/\alpha} - 1}{(\frac{q}{q_D})^{1/\alpha} - 1} \right) \left(\frac{c_1}{\sum_{k=1}^{n} c_k} \right) - 1 \right] D_{s,n}. \tag{100}$$

This is our proposed lower bound on minimum CCP regulatory capital requirements under cover-n DF. It is useful to compare this lower bound with the lower bound (54) derived under cover-one DF.

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